Semi-Analytic Modelling of Slot Waveguides in Silicon-Organic Hybrid Mach-Zehnder Modulators

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Abstract—A semi-analytic approach for modelling the distributed capacitance and the electrooptic confinement factor of the slot waveguide in a silicon-organic hybrid Mach-Zehnder modulator using the principle of conformal mapping is presented. The results show a deviation of less than 1.3% compared with numerical field simulations.

Index Terms—Mach-Zehnder modulator, silicon-organic hybrid

I. INTRODUCTION

Mach-Zehnder modulators (MZMs) built on silicon-organic hybrid (SOH) technology are used in optical communications channels and provide efficient modulation. Organic materials with electrooptic coefficients up to 1000 V/mm [1] combined with strong field confinement inside a slot waveguide, and high electric fields lead to voltage length products down to 0.41 V mm and electrooptic bandwidths up to 40 GHz [2]. Earlier works established an equivalent circuit model based on measurements [3]. However, a more simple method for finding the optimum parameters is desired in order to save time and effort. We present a semi-analytic model to calculate the distributed capacitance, as well as the electrooptic confinement factor, for the slot waveguide with the help of conformal mapping and the Schwarz-Christoffel transformation [4]. The results are then compared with numerical field simulations performed with CST Microwave Studio’s low frequency electrostatic solver.

II. THEORY AND RESULTS

This work investigates the basic structure of an SOH-MZM electrode shown in Fig. 1. A slot waveguide lies between gold conductors of a coplanar waveguide. The whole structure is covered by polymethylmethacrylate (PMMA), which carries the organic material with a high electrooptic coefficient. We also compare three different silicon-on-insulator technology parameter sets: Variant 1 (hWG = 250 nm, dBOX = 3 μm), variant 2 (hWG = 220 nm, dBOX = 2 μm), and variant 3 (hWG = 310 nm, dBOX = 2 μm).

A. Calculation of the Slot Capacitance

For the calculation of the slot capacitance we assume the silicon rails and slabs to be perfect electric conductors due to doping. The calculation area is limited by the edges of the silicon slabs. Based on [5] the distributed capacitance below the slot waveguide in a silicon-organic hybrid (SOH) technology are used in optical communications channels and provide efficient modulation. Organic materials with electrooptic coefficients up to 1000 V/mm [1] combined with strong field confinement inside a slot waveguide, and high electric fields lead to voltage length products down to 0.41 V mm and electrooptic bandwidths up to 40 GHz [2]. Earlier works established an equivalent circuit model based on measurements [3]. However, a more simple method for finding the optimum parameters is desired in order to save time and effort. We present a semi-analytic model to calculate the distributed capacitance, as well as the electrooptic confinement factor, for the slot waveguide with the help of conformal mapping and the Schwarz-Christoffel transformation [4]. The results are then compared with numerical field simulations performed with CST Microwave Studio’s low frequency electrostatic solver.

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z = w_{slot}/2
z = z_1 + jw_{WG}
|z_1| = d_{MM}
|z_2| = |z_1| + d_{MM}/2
|z_3| = |z_2| + w_{air}/2
|z_4| = |z_3| + jw_{WG}
|z_5| = |z_4| + w_{air}/2
|z_6| = |z_5| + j(h_{slab} + h_{PMMA})
|z_7| = z_5 + w_{lab}
|z_8| = |z_7| + j(h_{slab} + h_{PMMA})
|z_9| = |z_8| + w_{lab}
|z_{10}| = |z_9| + jh_{PMMA}
|z_{11}| = |z_{10}| + w_{lab}
|z_{12}| = |z_{11}| + jh_{PMMA}

Fig. 2. The cross section of the slot waveguide transferred into a coordinate system (z domain). (a) shows the initial polygon for the contribution of the air and (b) shows the initial polygon for the PMMA layer. Due to the finite height in (b) only the right side with an electric wall between z_1 and z_2 is mapped. and added up based on known methods of partial capacitance calculation [6].

Fig. 3. Semi-analytic and numerical values of the slot capacitance of variant 1 (blue), variant 2 (red), and variant 3 (green) in Fig. 5 show a maximum deviation of less than 1.3%. The semi-analytic calculation time per data point is less than 4 s, while the numerical simulation time is around 4 min with around $4 \times 10^5$ tetrahedrons and a maximum mesh edge length of about 300 nm.

B. Calculation of the Electrooptic Confinement Factor

The electrooptic confinement factor $\Gamma_{eo}$ describes the overlap between the electric field of the optical mode guided by the slot waveguide and the electric field $E_{appl}(x, y)$ of the applied electrode voltage $V$. The modulation efficiency is linear proportional to $\Gamma_{eo}$, which is defined as [7]

$$\Gamma_{eo} = \frac{n_{PMMA} w_{slot}}{2P_{opt}} \int_{V} \frac{E_{appl}(x, y)^2}{E_{appl}^{2}/2} \cos(\vartheta(x, y))^2 dx \, dy$$

where $E_{opt}$ is the electric field of the guided mode in the slot waveguide, $P_{opt}$ is the mode power, $Z_0$ is the free space characteristic impedance, and $\vartheta(x, y)$ is the angle between $E_{appl}(x, y)$ and $E_{opt}(x, y)$. By shifting the position of $z_1$ in Fig. 2(a) to the position of the magnetic wall ($w_{slot} + jw_{WG}/2$, it’s possible to calculate $E_{appl}(x, y)$ accurately. After mapping the slot waveguide’s cross section from the z domain over the intermediate w domain into a rectangle, it is easy to solve the Poisson’s equation for the electric potential, which is mapped back to the initial z domain. The electric potential of the upper half of the slot is mirrored into the lower half due to the magnetic wall at $y = h_{WG}/2$. $E_{appl}$ is then calculated by applying the gradient to the electric potential. In the end, the result is used to compute $\Gamma_{eo}$ with $E_{opt}$, which is simulated in Fimmwave. Fig. 4 shows the field profiles and verifies the accuracy of the proposed method, since the profiles match well.

III. CONCLUSION AND OUTLOOK

We presented an efficient semi-analytic method for calculating the capacitance of a slot waveguide as well as of the electrooptic confinement factor. The proposed method provides accurate results with a maximum deviation of 1.3% compared to numerical field simulations. By using the proposed semi-analytic method the computation time may be significantly reduced.

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REFERENCES