Passive and active slab waveguide mode analysis using transfer matrix method

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Abstract- **We present a general approach for numerical mode analysis of the multilayer slab waveguides using the Transfer Matrix Method (TMM) instead of the Finite Difference Frequency Domain (FDFD) method. TMM consists of working through the device one layer at a time and calculating an overall transfer matrix. Using the scattering matrix technique, we develop the proposed method for multilayer structures. We find waveguide modes for both passive and active slabs upon determinant analysis of the scattering matrix of the slab. Our proposed technique is more efficient and faster than other numerical methods.**

I. INTRODUCTION

Challenges in simulation of the electromagnetic devices have been reached to the design of time-variant (active) structures with various applications in nanophotonic and metasurface structures.

Here we present a technique to calculate slab waveguides modes using Transfer Matrix Method (TMM) semi-analytical algorithm [1] in which a device is represented as a stack of layers that are uniform (passive) or time-variant (active) in the longitudinal direction. Scattering matrices are calculated for each layer and are combined into a single overall matrix that describes propagation through the entire device. Free space gaps with zero thicknesses are inserted between the layers and the scattering matrices are made to relate fields that exist outside the layers, but directly on their boundaries [2]. Finally, we extract the modes in the slab waveguide structure using scattering matrix for both passive and active configurations.

II. PASSIVE SLAB WAVEGUIDE

First, we decide to analyze the guided modes in the waveguide of Fig. 1 sandwiched between two clads. The slab has a high refractive index of 2 between two identical clads with lower index of 1. The thicknesses of slab and each of the clads have been assumed equal to $3\lambda_0$ and $5\lambda_0$, respectively. Also, the incident wavelength (λ_0) is assumed to be 1µm. The electric wave propagates in the slab along x- direction and the polarization along z- direction will be as $\vec{E}(x,y,z) = \vec{A}(z)e^{-i\beta x}$, where $\vec{A}(z)$ is the amplitude profile and β is the propagation constant. In electromagnetic formulations, because of the structural uniformity, we assume $\frac{\partial}{\partial y} = 0$ and $\frac{\partial}{\partial x} = -j\beta$. Assuming $\beta = k_0 n_{eff}$ from ray tracing of propagation wave, we can reach the following equation:

$$\left[\frac{1}{k_0^2}\frac{d^2}{dz^2} + n^2(z)\right]E_y(z) = n_{eff}^2 E_y(z) \qquad (1)$$

By solving the above equation using finite difference frequency domain (FDFD) method [3], five first eigen-values can be reached as effective refractive indices of the guided modes which are travelling along x- direction as shown in Fig. 2. Blue lines are eigen-vectors which depict the modes.

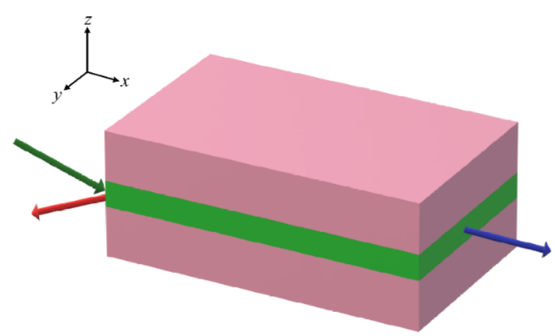

Fig. 1: Propagation along passive slab waveguide. The structure is consisting of high refractive index layer which is sandwiched between two clads with lower refractive indices.

Above analysis using FDFD method can be performed using TMM method but in different manner. In FDFD, the eigen-value problem calculates the modes directly, including the propagation constant as the eigen-value. Using TMM, we need to guess the value of propagation constant, calculate the scattering matrix and determine whether its determinant is zero. All propagation constants that cause the determinant of the global scattering matrix to be zero are related to guided modes. Here, we start TMM for the slab waveguide of Fig. 1 and try to sweep a range of values for n_{eff} and then calculate the determinant of the scattering matrix which is resulted from these refractive indices. Calculation of modes reduces to essentially a root-finding algorithm to determine the specific values of n_{eff}. To implement TMM, we assume the incident wavevector (k_{inc}) tackles with the first layer by elevation (θ) and azimuth (φ) angles and consists of three directional wavevectors as:

$$k_{inc} = c\, k_0 n_{inc} \begin{bmatrix} \sin(\theta)\cos(\varphi) \\ \sin(\theta)\sin(\varphi) \\ \cos(\theta) \end{bmatrix} = \begin{bmatrix} k_{inc,x} \\ k_{inc,y} \\ k_{inc,z} \end{bmatrix} \quad (2)$$

where n_{inc} and k_0 $(= \frac{\omega_{inc}}{c})$ are the refractive index of the incident environment and free space wavenumber, respectively. In our simulation codes, we have set $k_{inc,x} = k_0 n_{eff}$ and $k_{inc,y} = k_{inc,z} = 0$. Also, we have considered the least-squares problem to obtain $\min(\sum |f(x_i) - y_i|^2)$ based on the well-known Levenberg–Marquardt algorithm (LMA), where $f(x_i)$ is a nonlinear function like determinant of the scattering matrix obtained from the TMM algorithm and y_i is the desired result (obtained by setting the determinant equal to zero) [4]. We can perform root finding using cultural algorithm (CA) instead of LMA in structures with large number of layers [5]. The obtained results show that, TMM is pretty close to exact and faster than FDFD to find guided modes of the slab waveguide. We suggest the mode calculation using TMM, since the finite difference method tries to solve a very large amount of space outside the slab which consumes more memory and decreases the speed of calculations, especially for large and multilayer structures.

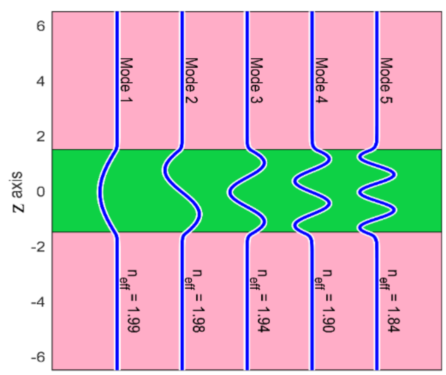

Fig. 2: First five eigen-vectors and eigen-values of the slab waveguide that show the guided modes of the travelling wave along propagation direction of x. The refractive indices of slab waveguide and identical clads are 2 and 1, respectively. All structure has a unique permeability.

III. Active Slab Waveguide

At this stage, we consider Fig. 1 with time-varying waveguide in which the time-modulated index of the core is presumed to have a sinusoidal behavior as $n(t) = n_s[1 + \delta \cos(2\pi f_m t)]$, such that the static refractive index n_s and modulation depth δ have the values of 2 and 0.01, respectively. By default, the relative magnetic permeability μ_r is assumed to be 1 and the excitation wavelength is considered equal to 1 μm. Also, the modulation frequency is set to ten percent of the incident frequency ($f_m = 0.1\, f_0$). We take cosine function instead of sine, since the cosine function has a peak in the core area and therefore the overall refractive index of the core material (waveguide) remains higher than both clads so that the condition of slab waveguide is not violated. The thicknesses of the slab and each of the clads are the same as section II.

Here, we set $k_{inc,x} = c k_0 n_{eff}$ from Eq. (2) and $k_{inc,y} = k_{inc,z} = 0$. Also, we assume the least-squares problem to obtain the zero point of the scattering matrix determinant obtained from the time-varying TMM (TTMM) simulation [6]. In the TTMM algorithm, we define time periodicity as $T = 2\pi/\Omega$, where $\Omega = 2\pi f_m$ is the modulation frequency with the frequency of f_m. Considering temporal variations, the periodicity is implemented to relative permittivity as $\varepsilon_r(t + uT) = \varepsilon_r(t)$. Also, angular frequency is expanded as $\omega(u) = \omega_{inc} - \frac{2\pi}{T} u$ with integer number of u.

In Table I, we have listed the unique modes. As seen in this table, the unique modes become fewer with increasing the temporal harmonics indices.

TABLE I

Number of temporal harmonics (u)	n_{eff}
3	2, 1.99, 1.97, 1.9, 1.86, 1.84, 1.83, 1.8, 1.79, 1.77, 1.75, 1.74, 1.68, 1.65, 1.64, 1.63
5	1.97, 1.9, 1.79, 1.77, 1.76, 1.75, 1.74, 1.64, 1.63,1.61
7	1.77, 1.74, 1.63
9	1.74, 1.63
11	1.74, 1.63

IV. Conclusion

In summary, with increasing the convergence of the purposed method in active state, precise modes could be calculated which their number is less than the modes analyzed in the passive slab waveguide. This can be related to the temporal behavior (variation) of the refractive index in the time-varying media which causes the number of propagation modes to be limited. The modulation frequency has the major effect on the amplitudes of effective modes in our analysis, so that we can increase/decrease or change the amplitudes of modes in active state (time-varying) versus various values of f_m. These modes that exhibit unusual dispersion relation may find applications in optical mixers, terahertz sources, and other optical devices [7].

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