

# Numerical Modeling and Experimental Validation of Tm- and Er-Doped Tellurite Microsphere Lasers

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**Abstract** — We report an efficient semi-analytical method for modeling of CW rare-earth ion-doped microlasers. We numerically simulated threshold pump powers and expected laser wavelengths for in-band pumped Tm- and Er-doped tellurite glass microspheres. With the increase of Q-factor, the laser wavelength grows smoothly for Tm and there are jumps from the C-band to the L-band for Er, which confirmed experimentally. The same method was used to study the possibility of multi-wavelength cascade lasing in Tm-doped microsphere pumped at 0.792  $\mu\text{m}$ . Depending on the pump power and Q-factor, single-wavelength lasing at 1.9  $\mu\text{m}$ , two-wavelength lasing at 1.9&1.5  $\mu\text{m}$  and at 1.9&2.3  $\mu\text{m}$ , and three-wavelength lasing at 1.9&1.5&2.3  $\mu\text{m}$  can be attained. The presented theoretical model can be used to simulate microlasers based on different matrices doped with different active ions.

## I. INTRODUCTION

Microsphere-based lasers are used as miniature photonic devices for basic science and many applications [1]. For such microlasers a gain medium and a cavity are the same device that is a microresonator with whispering gallery modes (WGMs) having high Q-factors and a small mode volume. Tellurite glasses are of great interest for the development of microsphere lasers [2]. But the development of their theory concedes to experimental achievements. Here we contribute to the development of the novel method of modeling of such devices and report simulation results validated experimentally.

## II. THE SEMI-ANALYTICAL METHOD OF MODELING

Microlasers based on rare-earth ions such as  $\text{Tm}^{3+}$ ,  $\text{Er}^{3+}$  etc., can be described by a 1st-order system of ordinary differential equations in time [3]. The system contains the coupled equations on intracavity pump ( $A_p$ ) and signal ( $A_{s,l}$ , where index  $l$  indicates the corresponding WGM) field envelopes together with rate equations describing the population densities  $n_i$  of energy levels (Fig. 1(a,b)) [3]. The traditional way to investigate such microlasers is numerical integration of a dynamical system (commonly by the Runge-Kutta method). A steady state CW solution can be found by modeling of the laser dynamics over a long time sufficient for decay of relaxation oscillations [3]. However, for a large number of WGMs located in the gain bands of a rare-earth ion, the dynamical system contains a lot of coupled equations [3], which can take significant time to solve, especially for many sets of parameters. For CW generation under CW pump, a solution can be found by solving a system of many nonlinear algebraic equations (of the order of the number of WGMs) obtained by taking  $d/dt=0$ . But we present another semi-analytical method allowing us to solve only a single nonlinear algebraic equation for  $n_1$  using exhaustive search for each

WGM (or a combination of WGMs belonging to different gain bands) and to find stable CW lasing regimes. After that,  $n_i$ ,  $A_p$ , and  $A_{s,l}$ , are expressed algebraically through  $n_1$ . Exhaustive search among all possible WGMs (no more than one WGM in each active band) is performed for each set of parameters and the conditions  $|A_p|^2$ ,  $|A_{s,l}|^2 > 0$  and  $0 < n_i < 1$  are verified. Among the remaining solutions we choose only the stable ones. If a WGM with order  $l^*$  is generated in the system only, then for all other WGMs with orders  $l \neq l^*$  the zero solution ( $A_{s,l} = 0$ ) must be stable, i.e. their increment must be negative. If there is no lasing, then all the increments must be negative. The direct Runge-Kutta simulation was also used to confirm the reliability of our method.

## III. NUMERICAL RESULTS

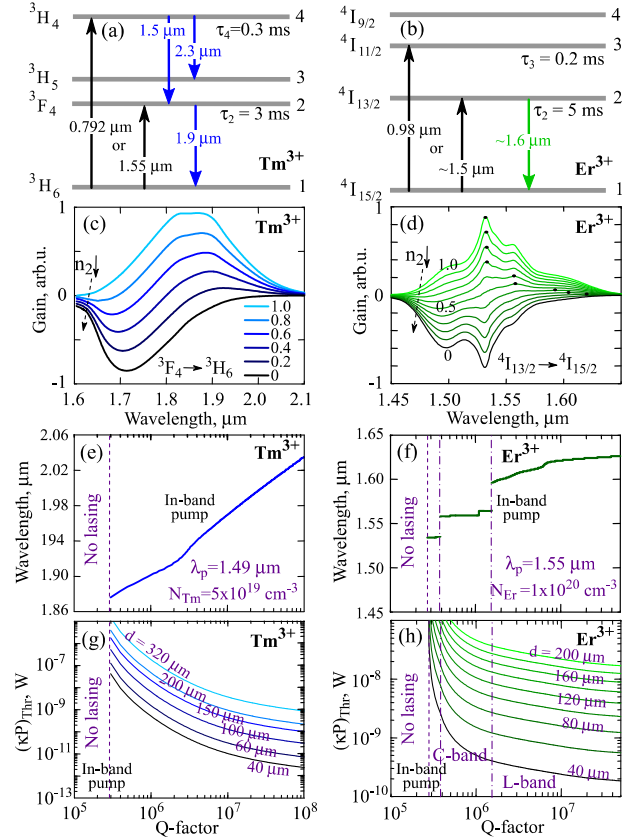


Fig. 1. Simplified schemes of energy levels of  $\text{Tm}^{3+}$  (a) and  $\text{Er}^{3+}$  (b). Normalized gain function proportional to  $(\sigma_{em}n_2 - \sigma_{abs}n_1)$  for  $\text{Tm}^{3+}$  (c) and  $\text{Er}^{3+}$  (d). Calculated laser wavelengths for in-band pumped microspheres doped with  $\text{Tm}^{3+}$  (e) and  $\text{Er}^{3+}$  (f). Threshold values of  $\kappa P_{\text{thr}}$  (pump powers multiplied by coupling coefficients) for different diameters of microspheres doped with  $\text{Tm}^{3+}$  (g) and  $\text{Er}^{3+}$  (h).

At first, we analyzed microlaser with in-band CW pump at the  ${}^3\text{H}_6 \rightarrow {}^3\text{F}_4$  and  ${}^4\text{I}_{15/2} \rightarrow {}^4\text{I}_{13/2}$  energy transitions for  $\text{Tm}^{3+}$ -doped and  $\text{Er}^{3+}$ -doped tellurite microsphere (Fig. 1(a,b)), respectively. Figure 1(c,d) demonstrate the gain functions proportional to  $(\sigma_{\text{em}}n_2 - \sigma_{\text{abs}}n_1)$  vs  $n_2$ , where  $\sigma_{\text{em,abs}}$  are the emission and absorption cross-sections. The wavelength corresponding to the maximum of the gain function shifts smoothly for  $\text{Tm}^{3+}$ , but there are jumps for  $\text{Er}^{3+}$ , which explains the behavior of expected laser wavelength dependence on Q-factors (Fig. 1(e,f)). The laser wavelength does not depend on microsphere diameter  $d$ , but the laser threshold depends on both, Q-factor and  $d$  (Fig. 1(g,h)) [4].

Next, we study the possibility of multi-wavelength cascade lasing in  $\text{Tm}^{3+}$ -doped microsphere pumped at 0.792  $\mu\text{m}$  (Fig. 1(a)). Depending on the pump power and Q-factor, single-wavelength lasing at 1.9  $\mu\text{m}$ , 2-wavelength lasing at 1.9&1.5  $\mu\text{m}$  and at 1.9&2.3  $\mu\text{m}$ , and 3-wavelength lasing at 1.9&1.5&2.3  $\mu\text{m}$  can be attained (Fig. 2(a)). The dependences of the output laser power on the pump power are plotted in Fig. 2(b,c) for  $Q=3 \cdot 10^7$  and  $Q=10^6$ , respectively. The 1st laser threshold is reached for the wave at  $\sim 1.9$   $\mu\text{m}$  for  $Q > 3 \cdot 10^5$ . For a high enough Q-factor, the 2nd threshold is achieved at  $\sim 1.5$   $\mu\text{m}$  or at  $\sim 2.3$   $\mu\text{m}$ .

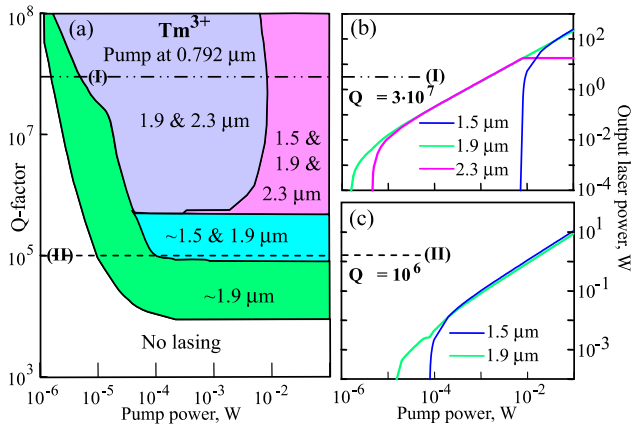


Fig. 2. (a) Numerically simulated diagram of generation regimes in  $\text{Tm}^{3+}$ -doped microsphere with  $d=50$   $\mu\text{m}$  pumped at 0.792  $\mu\text{m}$ . Calculated output laser powers as functions of pump power for  $Q=3 \cdot 10^7$  (b) and  $Q=1 \cdot 10^6$  (c).

#### IV. EXPERIMENTAL RESULTS

To verify our theoretical results, we also performed two experimental series on laser generation using tellurite microspheres doped with 1)  $\text{Tm}^{3+}$  ions and 2)  $\text{Er}^{3+}$  ions under in-band pump by the same tunable C-band CW laser (Fig. 3(a)). High-quality tellurite glasses were fabricated using the previously developed technique [4-6]. Microspheres were made of single-index fibers based on these glasses using a resistive micro-heater [4]. We fabricated a microsphere with diameter  $d = 320$   $\mu\text{m}$  doped with  $\text{Tm}^{3+}$  ions with concentration  $N_{\text{Tm}} = 5 \cdot 10^{19}$   $\text{cm}^{-3}$  and a microsphere with diameter  $d = 80$   $\mu\text{m}$  doped with  $\text{Er}^{3+}$  ions with concentration  $N_{\text{Tm}} = 1 \cdot 10^{20}$   $\text{cm}^{-3}$ . Measured Q-factors were of the order of  $10^6$ . We used a tapered fiber made of a standard silica SMF28e [7] for pumping of the microsphere and outcoupling of the generated laser radiation. The laser thresholds were a few mW before the tapered fiber in both cases. A composite spectrum of the transmitted pump (in dB scale) and the generated laser wave

(in linear scale) in  $\text{Tm}^{3+}$ -doped microsphere is plotted in Fig. 3(b). Lasing was attained at  $\sim 1.9$   $\mu\text{m}$ , which agrees with the numerical results shown in Fig. 1(e), where for  $Q=10^6$ , the expected laser wavelength is the same. For  $\text{Er}^{3+}$ -doped microsphere, by changing the parameters of the system (the distance between the microsphere and the tapered fiber and the pump power) we obtained different regimes of lasing (Fig. 3(c,d)). We observed that at smaller distances ( $\sim 0.5$   $\mu\text{m}$ ) between the taper and the microsphere, i.e. at larger coupling and lower loaded Q-factor, lasing tends to a shorter wavelength in the C-band (Fig. 3(c)). When the distance between the taper and the microsphere was larger ( $\sim 0.9$   $\mu\text{m}$ ), i.e. the loaded Q-factor was higher, lasing occurred in a longer-wavelength region in the L-band (Fig. 3(d)). This jump of the laser wavelength from the C-band to the L-band agrees with the numerical results presented in Fig. 1(f).

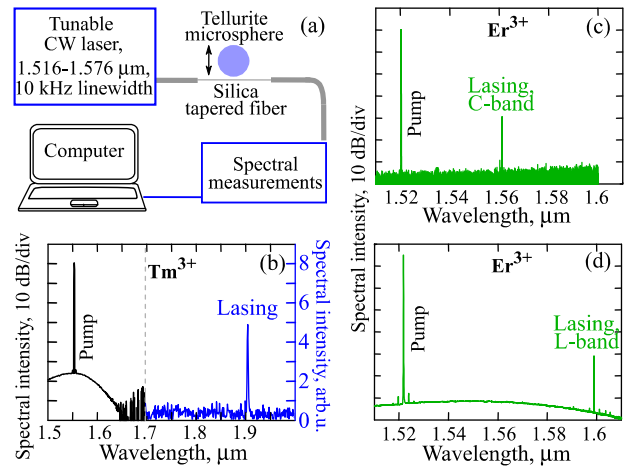


Fig. 3. (a) Simplified experimental scheme. (b) Experimental spectra attained in  $\text{Tm}^{3+}$ -doped microsphere. Experimental spectra corresponding to C-band lasing (c) and L-band lasing (d) in  $\text{Er}^{3+}$ -doped microsphere. Distances between  $\text{Er}^{3+}$ -doped microsphere and taper are  $\sim 0.5$   $\mu\text{m}$  (c) and  $\sim 0.9$   $\mu\text{m}$  (d).

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