Modelling of photon recycling in optoelectronic devices using a transfer matrix method

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***Abstract*—In this work we present a generalized transfer matrix method to study the effect of the photon recycling on the performance of solar cells. Photon recycling increases the charge carrier concentration in solar cells, resulting in an increase of the open circuit voltage (V_{oc}). The model is based on the transfer matrix method (TMM), taking into account internal sources representing the emission from radiative recombination.**

I. Introduction

The transfer matrix method (TMM) is one of the most used models for one-dimensional optical modeling of devices [1], [2]. It provides a suitable modeling for the calculation of the electromagnetic field confinement considering interference and diffraction effects along the optical path and avoiding an increase of the computational efforts.

Starting from the TMM model with an external radiation, in this work we present an optical model that allows to take into account the effects of incoherent sources within the device. In this way it is possible to represent the self-absorption of the photons emitted internally from the radiative recombination. This effect is called photon recycling [3] and it increases the charge carrier concentration, resulting in an increase of the open circuit voltage (V_{oc}). This model allows to study optoelectronic devices both for photovoltaic applications and for light emitting devices like light emitting diodes (LEDs).

II. Modeling

We implemented a transfer matrix method taking into account an internal source as shown in Fig. 1. Let us assume that the device can be divided in homogeneous layers with flat and parallel interfaces. First (N_0) and last (N_{z+1}) layers are considered semi-infinite. The notation in black refers to an external radiation, the red one refers to an internal source. For each layer of the structure the electromagnetic field is decomposed in a *forward* E^f and *backward* E^b travelling waves. E^f_{0R} and E^b_{0R} are the forward and backward components just before the first interface N_0, $E^f_{(z+1)L}$ and $E^b_{(z+1)L}$ are the components after the last interface N_{z+1}. Using the scattering-matrix formalism, we can relate these four components as shown in (1):

$$\begin{bmatrix} E^f_{0R} \\ \\ E^b_{0R} \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} E^f_{(z+1)L} \\ \\ E^b_{(z+1)L} \end{bmatrix} \tag{1}$$

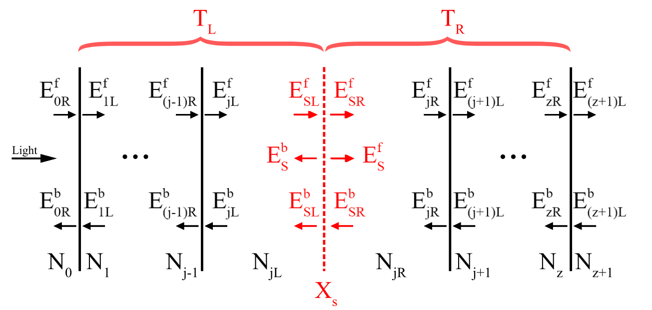

Fig. 1. Scheme of the transfer matrix method, the forward E^f and backward E^b electric field components are shown. The black notation refers only to an external radiation from left to right. In order to consider only the internal source the red notation is used. X_S in the position of the internal source.

where **T** is the *transfer matrix* defined as the product $\boldsymbol{M}_j$ and $\boldsymbol{I}_{j,j+1}$. The first one represents the propagation of the electric field through the layer j. It relates the electric field at the left interface to the one at the right interface. The matrix $\boldsymbol{M}_j$ is defined as

$$\boldsymbol{M}_j = \begin{bmatrix} e^{-ik_j t_j} & 0 \\ 0 & e^{ik_j t_j} \end{bmatrix} \tag{2}$$

where t_j and k_j are the thickness and the propagation constant of layer j respectively. The interface matrix $\boldsymbol{I}_{j,j+1}$ describes the propagation of the electric field at the interface between adjacent layers in terms of reflection and transmission coefficients. Considering the layer j and the next $i = j + 1$, it is defined as

$$\boldsymbol{I}_{ji} = \frac{1}{t_{ji}} \begin{bmatrix} 1 & r_{ji} \\ r_{ji} & 1 \end{bmatrix} \tag{3}$$

where r_{ji} and t_{ji} are the reflection and transmission Fresnel coefficients. In order to represent the internal emission from radiative recombination we have to take into account an internal source as depicted with red notation in Fig. 1. X_S represents the position of the internal source in layer j, E^f_S and E^b_S are the forward and backward components of the electric field of the internal emission, respectively. At the source position we can relate the components of the electric field on the left side of the source to the ones at the right side as shown in (4) and (5):

$$E^f_{SR} = E^f_{SL} + E^f_S \tag{4}$$

$$E^b_{SL} = E^b_{SR} + E^b_S \tag{5}$$

Using the scattering-matrix formalism it is possible to relate the electric field on the left and on the right side of the source defining the transfer matrix as $\mathbf{T}_L$ and $\mathbf{T}_R$ for the left and right side, respectively. In order to calculate the light out-coupling from a source inside the device we can assume that there are no electric field components entering the device, so that $E^f_{0R} = 0$ and $E^b_{(z+1)L} = 0$. Thus we obtain

$$\begin{bmatrix} -T_{11} & 0 \\ -T_{21} & 1 \end{bmatrix} \begin{bmatrix} E^f_{(z+1)L} \\ E^b_{0R} \end{bmatrix} = \boldsymbol{T}_L \begin{bmatrix} -E^f_S \\ E^b_S \end{bmatrix} \tag{6}$$

where $\boldsymbol{T}_L$ is the transfer matrix to the left of the source, T_{11} and T_{12} are the coefficient of the product $\boldsymbol{T}_L \cdot \boldsymbol{T}_R$. From (6) it is possible to calculate the external electric field components taking into account the emission of the internal source. For simplicity, the emission is considered equally distributed on the whole solid angle. The resulting intensity obtained from the TMM with external source and the one with internal sources are summed up incoherently, since in the shown examples we assume internal emission being due to only spontaneous emission.

Regarding the electrical model, we assume for the present examples the electron and hole concentrations to be constant along the whole device, i.e. we do not need to solve a transport model but can impose the splitting of the quasi Fermi levels, instead.

III. Results and Discussion

We simulated a GaAs solar cell with three different structures. The first one is made of a GaAs layer with planar front and rear surfaces. The layer thickness is 180 nm. In the second architecture a perfectly reflecting mirror on the rear surface is added. In the third one a distributed Bragg Reflector (DBR) on the front surface is added, maintaining the back mirror. The DBR has a stop band centered at the emission wavelength of $\lambda_0 = 873$ nm and is made of Si_3N_4/SiO_2. The TMM model with external radiation was used considering incident sunlight according to the Air Mass 1.5 Spectrum. Figure 2 (a) shows the intensity of the electric field obtained with the external source for all geometries. The TMM model with internal source was used to take into account the emission of the radiative recombination within the device. We considered an internal emission centered at $\lambda_0 = 873$ nm corresponding to the energy gap of GaAs, neglecting the spectral broadening of the emission spectrum. Figure 2 (b) shows the intensity of the electric field obtained from the internal emission only. The JV curves are presented in Fig. 2 (c). While the effect of the back mirror is mostly an increase of the short circuit current due to the longer effective optical path, the DBR in addition leads to an increase of the charge carrier concentration due to the confinement of the electromagnetic field at λ_0, resulting in an increase of V_{oc}.

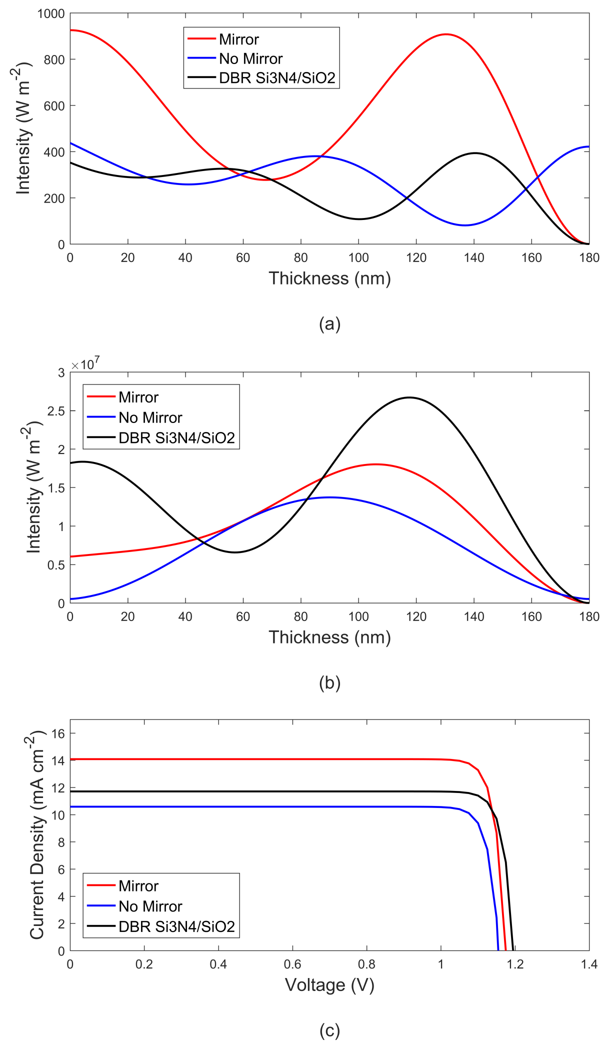

Fig. 2. Intensity of the electric field within the device for the TMM model with external radiation (a) and homogeneously distributed internal source (b). (c) J-V curves of the solar cell. The blue and red lines correspond to structure without and with mirror, respectively. The black line refers to the structure with BR.

References

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