A Functional Mapping for Passively Mode-Locked Semiconductor Lasers

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Abstract—We present a modern approach for the analysis of passively mode-locked semiconductor lasers that allows for efficient parameter sweeps and time jitter analysis. It permits accessing the ultra-low repetition rate regime where pulses become localized states. The analysis including slow (e.g. thermal) processes or transverse, diffractive dynamics becomes feasible. Our method bridges the divide between the phenomenological, yet highly efficient, pulse iterative model that is the Haus master equation, and the more involved first principle descriptions relying on time delayed equations. Reductions of the simulation times and of the memory footprint up to two orders of magnitudes are demonstrated.

Index Terms—Semiconductor laser, Vertical Cavity Surface Emitting Lasers

I. INTRODUCTION

Generation of low repetition rate picosecond pulses is of paramount importance for a number of applications [1], [2]. Passive Mode-locking (PML) of semiconductor lasers is a most promising method, although it still represents an experimental and a theoretical challenge, see [3] for a recent review. The Haus master equation [4] is an efficient and widely used approach to study PML. It consists in restricting the analysis of the field to a small temporal interval around the pulse. Yet this method, when applied to a particular design, provides only qualitative predictions due to the many simplifying hypothesis involved. How to derive *the* Haus equation, for a specific laser design, is also an open question. On the other hand, first principle modeling allows representing the full dynamics of both unidirectional and bidirectional cavities as either Delay Differential Equations (DDEs) [5] or Delay Algebraic Equations (DAEs) [6], respectively. Such models have been applied successfully to the study of PML with saturable absorber (SA), and were extended to describe photonic crystals [7], external optical feedback [8], coherent optical injection [9], nonlocal imaging conditions [10] and localized structures (LSs) [11].

II. RESULTS

We illustrate the idea of the functional mapping (FM) [12] using the DDE model of [5] that considers unidirectional propagation in a ring laser. The equations for the field amplitude A, the gain G and the absorption Q read

$$\frac{\dot{A}}{\gamma} = -A + Y(t - \tau), \quad (1)$$

$$\dot{G} = \Gamma(G_0 - G) - e^{-Q}(e^G - 1)|A|^2, \quad (2)$$
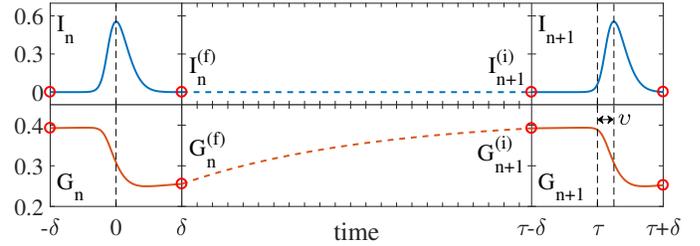
$$\dot{Q} = Q_0 - Q - s(1 - e^{-Q})|A|^2, \quad (3)$$

Figure 1. Temporal profile of the intensity $I_n = |A_n|^2$ and of the gain G_n at the n-th and $n+1$-th round-trips. After the emission of the pulse I_n and the ensuing gain depletion, the so-called fast stage (solid lines), the gain recovers until the next round-trip while the field is vanishing (dashed lines). Knowing the final value of $G_n^{(f)}$ in the interval $z \in [-\delta, \delta]$, one can deduce the initial gain value at the next round-trip $G_{n+1}^{(i)}$. The central panel is not up to scale and can be several orders of magnitude larger than the outer panels.

with

$$Y(t) = \exp\left[\frac{1 - i\alpha}{2}G(t) - \frac{1 - i\beta}{2}Q(t)\right]A(t), \quad (4)$$

where G_0 is the pumping strength, $\Gamma = \tau_g^{-1}$ the gain recovery rate, Q_0 the value of the unsaturated losses which determines the modulation depth of the SA and s the ratio of the saturation energy of the gain and of the SA sections. We define κ as the intensity transmission of the output mirror, i.e., the fraction of the power remaining in the cavity after each round-trip. In Eqs. (1-3) time has been normalized to the SA recovery time that we assume to be $\tau_q = 20\,\text{ps}$. The linewidth enhancement factor of the gain and absorber sections are noted α and β, respectively. In addition, γ is the bandwidth of the spectral filter whose central optical frequency has been taken as the carrier frequency for the field. This spectral filter may (coarsely) represent, e.g., the resonance of a VCSEL [10]. If not otherwise stated $(\kappa, \alpha, \beta, s) = (0.8, 2, 0.5, 30)$, and $Q_0 = 0.3$ which corresponds to modulation of the losses of $\sim 26\,\%$. We also set $\gamma = 10$ and $\Gamma = 0.04$, leading to a gain bandwidth full width at half maximum of $160\,\text{GHz}$ and $\tau_g = 500\,\text{ps}$.

We wrote Eq. (1) in a form that makes apparent that the forcing field $Y(t - \tau)$ defined in Eq. (4) is a nonlinear function that is known over a whole interval of duration τ. Since G and Q are functions of A, Y involves *only* the past values of the field, i.e., $Y(t - \tau) = g[A(t - \tau)]$. Knowing the forcing term Y, Eq. (1) can be solved for A over an interval of duration τ. Integrating Eqs. (1-4), not over the whole round-trips, but only in a selected time interval in

the pulse vicinity, is at the core of our method. We define the field and carrier profiles at the n-th round-trip as $A_n(z)$ and $(G_n, Q_n)(z)$. For clarity, we set in Fig. 1 the pulse at the origin of time at the n-th round-trip. Next, we define a small interval of duration 2δ and a local time $z \in [-\delta, \delta]$. Finally, we impose a condition on the waveform A_n: it is a pulse of duration τ_p asymptotic to $A = 0$ if $\delta \gg \tau_p$. Under these approximations, one can solve Eq. (1) using standard integration techniques, e.g., Runge-Kutta method, at the next round-trip, using the following sequence $(A_n, G_n, Q_n) \rightarrow Y_n \rightarrow (A_{n+1}, G_{n+1}, Q_{n+1})$. Doing so corresponds to writing a functional mapping $A_{n+1} = F(A_n)$. The remainder of the dynamics during the round-trip of duration $r = t - 2\delta$, see central panel in Fig. 1, in which the field is vanishing can be found by solving Eqs. (2,3) analytically in the absence of stimulated emission, the so-called slow stage of PML. As such, $G_{n+1}^{(i)} = G_n^{(f)}\chi + G_0(1 - \chi)$ with $\chi = \exp(-\Gamma r)$ and similarly for $Q_{n+1}^{(i)}$. Solving analytically the slow stage allows to fully cancel the stiffness inherent to the multiscale nature of PML which is exceptionally useful in the long delay limit $\tau \gg \tau_g$. The speedup of our method is equal to the ratio of the actual integration domain 2δ and of the full round-trip τ, i.e., $m = \tau/(2\delta)$. Taking a domain of duration $2\delta = 5\tau_p$, a pulse-with of $\tau_p = 1\,\mathrm{ps}$ at a repetition rate of $\tau^{-1} = 1\,\mathrm{GHz}$, yields a speedup $m = 200$, i.e., a 24 hour simulation with, e.g., slow thermal effects or transverse diffraction could potentially be achieved in a few (~ 7) minutes.

We conclude our analysis by showing how the FM can be used for the simulation of broad area MIXSEL system described by the DAE model of [6], [10]. The model for the intra-cavity field E, gain N_1 and absorption N_2 reads

$$\dot{E} = [(1 - i\alpha_1)N_1 + (1 - i\alpha_2)N_2 - 1 + i\Delta_\perp]E + hY \quad (5)$$
$$\dot{N_1} = \gamma_1(J_1 - N_1) - N_1|E|^2, \quad (6)$$
$$\dot{N_2} = \gamma_2(J_2 - N_2) - sN_2|E|^2. \quad (7)$$

while the relation linking the intra-cavity (E) and external cavity (Y) fields is, after proper normalization,

$$Y(t) = \eta[E(t - \tau) - Y(t - \tau)] \quad (8)$$

with η the external mirror reflectivity, see [10] for more details. For the sake of simplicity, we consider diffraction in a single transverse dimension, making the problem two-dimensional and allowing for easier multi-parameter bifurcation analysis. We concentrate on the spatio-temporal localization regime where the field coalesces into a a spatio-temporal dissipative soliton, also called a Light Bullet (LB) [13], [14]. Figure 2 (a) shows a two-dimensional bifurcation diagram for the case $\alpha_j = 0$ as a function of the forward and reverse bias in the gain and absorber sections J_1 and J_2. Finding the region of stability in Fig. 2a) required 24 hours on a standard PC using the FM, instead of several months integrating the full DAE system. Figure 2 (b) depicts the spatio-temporal LB profile obtained with the FM method.
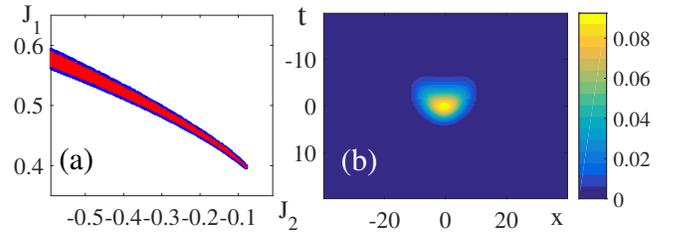

Figure 2. (a) Two-dimensional bifurcation diagram showing the region of stable existence of the LBs of the DAE model (5-7) as a function of the reverse bias in the gain and absorber sections J_1 and J_2. (b) Spatio-temporal profile of the field found with the FM with $(J_1, J_2) = (0.498, -0.336)$. Other parameters are $(\alpha_1, \alpha_2, h, \gamma_1, \gamma_2, s, \eta) = (0, 0, 2, 0.003, 0.1, 30, 0.5)$.

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