Third Order Dispersion in Optical Time Delayed Systems: The case of Mode-Locked Vertical External-Cavity Surface-Emitting Lasers

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***Abstract*—Time-delayed dynamical systems materialize in situations where distant, point-wise, nonlinear nodes exchange information that propagates at a finite speed. However, they are considered devoid of dispersive effects, which are known to play a leading role in pattern formation and wave dynamics. We show how dispersion may appear naturally in delayed systems and we exemplify our result by studying theoretically and experimentally the influence of third order dispersion in a system composed of coupled optical micro-cavities. Dispersion induced pulse satellites emerge asymmetrically and destabilize the mode-locking regime.**

***Index Terms*—Semiconductor laser, Vertical Cavity Surface Emitting Lasers**

I. Introduction

Delayed dynamical systems (DDSs) describe a large number of phenomena in nature and they exhibit a wealth of dynamical regimes such as localized structures (LSs) [1], fronts and chimera states. A fertile perspective lies in their interpretation as spatially extended *diffusive* systems which holds in the limit of long delays [2]. A strong limitation of DDSs for this aim comes from the difficulty of taking into consideration chromatic dispersion. As such, DDSs description of wave propagation is limited to low dispersive media and/or narrow-spectrum signals. Nevertheless, chromatic dispersion plays a leading role in many phenomena occurring during wave evolution [3]. Beyond second order dispersion (SOD), third order dispersion (TOD) is the lowest order non-trivial parity symmetry breaking effect, which leads to convective instabilities [4] and drifts.

In this work [5] we show that SOD and TOD appear naturally when modeling a large class of optical DDSs including Vertical External-Cavity Surface-Emitting Lasers (VECSELs), Mode-Locked Integrated External-Cavity Surface Emitting Laser (MIXSELs), laser resonators with intracavity Gires-Tournois etalons, and, more generally, distantly-coupled laser cavities. To illustrate our general result, we focus on a mode-locked VECSEL and we theoretically predict how TOD destabilizes mode-locked pulses. In particular, the passive mode-locking of VECSELs is based upon the dynamics of coupled micro-cavities. We show that the high reflectivity of the bottom mirror in these micro-cavities, combined with strong delayed feedback, induces TOD which generate a peculiar waveform: An infinite sequence of decaying satellites accumulate in front of the pulse leading edge. We show that the unbalanced micro-cavities act as resonant Gires-Tournois interferometers that are a source of third order dispersion inducing the satellites. We predict a new kind of instability for the pulse train in which a satellite may become unstable and grow into a fully developed pulse which, due to the induced gain depletion, eventually kills the pulse from which it originates, leading to a low frequency quasi-periodic dynamics. Experimental evidences are provided showing good agreement with model predictions.

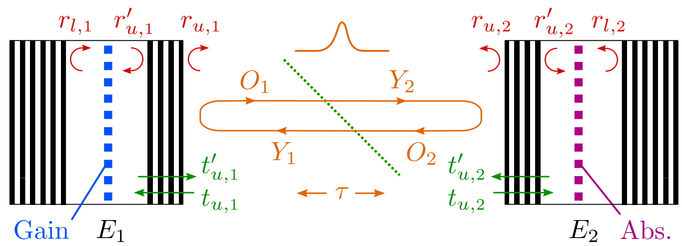

Figure 1. A schematic of the coupled cavities configuration. E_j denote the intra-cavity fields, $j = 1, 2$. The output and input fields in the external cavity are represented by O_j and Y_j, respectively. The time of flight in the cavity is τ.

II. Results

The setup of our VECSEL setup is depicted in Fig. 1. Our theoretical approach follows the method developed in [6], [7], [8]. The dynamical model for the intra-cavity fields E_j and population inversions N_j reads

$$\begin{aligned}
\kappa_1^{-1}\dot{E}_1 &= \left[(1-i\alpha_1)\,N_1-1\right]E_1+h_1Y_1, &(1)\\
\kappa_2^{-1}\dot{E}_2 &= \left[(1-i\alpha_2)\,N_2-1+i\delta\right]E_2+h_2Y_2, &(2)\\
\dot{N}_1 &= \gamma_1\left(J_1-N_1\right)-\left|E_1\right|^2N_1, &(3)\\
\dot{N}_2 &= \gamma_2\left(J_2-N_2\right)-s\left|E_2\right|^2N_2. &(4)
\end{aligned}$$

The indexes $j = 1, 2$ denote a gain mirror and a semiconductor saturable absorber mirror (SESAM), respectively. The photon lifetimes are κ_j^{-1}, the detuning between the two cavities is δ, α_j denote the linewidth enhancement factors

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and γ_j^{-1} are the population lifetimes. The forward and reverse bias of the gain and the saturable absorber are noted J_1 and J_2, respectively. The lasing threshold is defined as $J_{\rm th}$ and emission occurs for $J_1 \geq J_{\rm th}$. The ratio of the gain and absorber saturation intensities is s. The fields injected into the micro-cavities are Y_j with a coupling factor given by $h_j = (1+|r_{l,j}|)(1-|r_{u,j}|)/(1-|r_{u,j}r_{l,j}|)$ with $r_{u,j}$ and $r_{l,j}$ the upper and lower distributed Bragg mirror reflectivities (in amplitude). The cavity outputs consist in a superposition between the reflected and emitted fields and reads, after proper normalization, $O_j = E_j - Y_j$. As such, considering the time of flight between the two micro-cavities τ as well as the presence of the beam sampler with transmission amplitude $t_{\rm bs}$, we find that the mutual injection between the two cavities is given by two delay algebraic equations

$$Y_1(t) = O_2(t-\tau) = t_{\rm bs}\left[E_2(t-\tau) - Y_2(t-\tau)\right], \quad (5)$$

$$Y_2(t) = O_1(t-\tau) = t_{\rm bs}\left[E_1(t-\tau) - Y_1(t-\tau)\right]. \quad (6)$$

In the passive mode-locking regimes where short optical pulses are generated by the interplay between the nonlinear dynamics of gain and saturable absorption in the two cavities, in some situations, in particular for high bias current when the background zero intensity solution becomes weakly stable, one of the satellites, generally the one of highest intensity closest to the main pulse, can become linearly unstable and grow into a fully developed pulse. In this case, the newborn pulse will deprive the original pulse of gain, due to the finite recovery time of the carriers. This behavior can be interpreted as an *asymmetrical* soliton explosion [9], [10], [11], [12]. Such a situation leads to the dynamics depicted in Fig. 2. The individual pulse dynamics are most readily observed using a so-called space-time diagram as in Fig. 2(a), where one can see the details of the pulse evolution over many round-trips. Here the growth of the satellites in front of the pulse, and the subsequent death of the original pulse is most visible and leads to an apparent motion of the pulse to the left. This ultrafast dynamics can be partially blurred by a photo-detector having a limited bandwidth, but it remains visible, see Fig. 2(b). The optical spectrum is depicted in Fig. 2(c) where one can observe the evolving asymmetrical tail of the bluest side. A slightly different parameter set leads to deformed satellite explosions that resemble to an oscillation of the pulse position, see in Fig. 2(d). This oscillation remains visible even blurred by a photo-detector as shown in Fig. 2(e). Oscillation of the optical spectrum can also be observed, cf. Fig. 2(f). In all cases, one notes the very long time scale of this dynamics that corresponds to the slow evolution of the satellite from one round-trip towards the next under the influence of nonlinearity, drift and third order dispersion.-

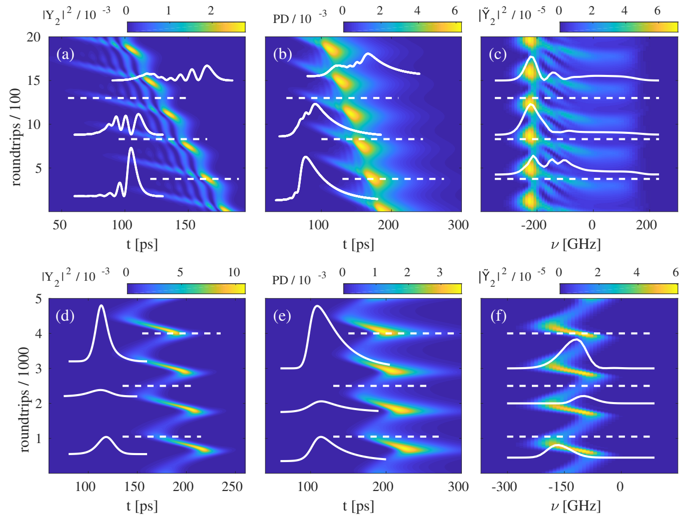

Figure 2. (a,d) Space-time diagrams for the intensity of a passively mode-locked VECSEL. (b,e) Low pass filtered time trace @10 GHz in order to emulate finite Photodetector bandwidth. (c,f) Single shot optical spectrum. (a,b,c) and (d,e,f) correspond to two different parameters set, and the satellites pulsating on the front of the pulse can take different shapes as a function of the effective amount of TOD

ACKNOWLEDGEMENTS

C.S. and J.J. acknowledge the financial support of the MINECO Project MOVELIGHT (PGC2018-099637-B-100 AEI/FEDER UE). S.G. acknowledges the PRIME program of the German Academic Exchange Service (DAAD) with funds from the German Federal Ministry of Education and Research (BMBF).

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