Reflection Spectra Analysis and Optimization of Phase-modulated Waveguide-grating Reflectors

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Abstract- **Phase-modulated waveguide gratings are promising reflective optical elements for widely tunable laser applications. We present an analysis on their comb-like reflectivity response and address the impact of phase-shifts on the number of large reflection peaks, their envelope and on the effective grating coefficient. Utilizing a robust phase-modulation approach, this paper shows a quasi-stochastic numerical optimization method to control the reflectivity comb effectively.**

Index Terms- DBR, grating, phase modulation, reflectivity comb

I. INTRODUCTION

Thermally tuned semiconductor lasers are key devices for digital coherent systems. In such applications even wider tuning range and narrower linewidth are required, while maintaining low power consumption to enhance the effectiveness and transmission capacity. Monolithically integrated distributed Bragg reflector (DBR) lasers have made considerable progress towards these requirements. In order to employ Vernier-effect based tuning, DBRs should have comb-like reflection spectra [1]. Sampled or chirped-sampled grating (SG) DBR realizes this feature by rectangular amplitude modulation of the grating [2,3], while phase-modulated grating (PG) employing several discrete phase-shifts in a repeating pattern [4].

In this paper we have derived and compared the reflection spectra of gratings. The analysis shows that PG-DBRs are promising optical elements due to their large effective grating coefficient and customizable reflectance spectra. We have demonstrated that the finite number of peaks and the envelope of its reflectivity comb can be controlled effectively by a quasi-stochastic numerical optimization method.

II. FOURIER ANALYSIS OF MODULATED GRATINGS

The analyzed phase-modulated gratings have Λ pitch size and contain multiple (N) periods with identical Λ_S periodicity. The discrete $\Lambda/2$ phase shifts (extra teeth or holes) are repeated in all of the N periods. Let us assume M shifts within a period: $x_1 < x_2 < \ldots < x_j < \ldots < x_M$ by definition $x_{M+1} = N\Lambda_S + x_1$, and so on. No restrictions are applied between the shifts, like antisymmetric phases [4].
The reflection spectrum of a grating $\left(F(k)\right)$ can be derived from the Fourier transform of the modulation function $\left(f(x)\right)$:

$$f_P(x) = \begin{cases} \pm 1, if\ \left(n-1+x_j\right)\Lambda_S < x < \left(n-1+x_{j+1}\right)\Lambda_S \\ 0, if\ x<0\ or\ x>N\Lambda_S \end{cases} \quad n = 1,\ldots,N \tag{1}$$

<u>Even number of phase-shifts (*M*):</u>

$$F_p(k) = \sum_{n=1}^{N-1} \sum_j (-1)^j \left[\frac{exp(ikx)}{ik}\right]_{x=(n-1+x_j)\Lambda_S}^{x=(n-1+x_{j+1})\Lambda_S} \tag{2}$$

$$\left|F_p(\hat{k})\right|^2 = \Lambda_S^2 \left[\frac{1-cos(N\hat{k})}{1-cos(\hat{k})}\right] \frac{\sum a_{jl} \cos\left(\hat{k}(x_j-x_l)\right)}{\hat{k}^2/2} \tag{3}$$

<u>Odd number of phase-shifts (*M*):</u>

$$F_p(k) = \sum_{n=1}^{N-1} (-1)^{(n-1)} \sum_j (-1)^j \left[\frac{exp(ikx)}{ik}\right]_{x=(n-1+x_j)\Lambda_S}^{x=(n-1+x_{j+1})\Lambda_S} \tag{4}$$

$$\left|F_p(\hat{k})\right|^2 = \Lambda_S^2 \left[\frac{1-(-1)^N cos(N\hat{k})}{1+cos(\hat{k})}\right] \frac{\sum a'_{jl} \cos\left(\hat{k}(x_j-x_l)\right)}{\hat{k}^2/2} \tag{5}$$

All power spectra contain a factor $\frac{1\mp cos(N\hat{k})}{1\mp cos(\hat{k})}$, where $\hat{k} = k\Lambda_S$. This defines the comb behavior of reflectivity with $1/\Lambda_S$ scaled peak separation. The magnitude of central frequency (k = 0) is proportional to N^2, and full width of peak scales with $1/N$.

For even numbers of M, the successive periods constructively interfere, and the (±1)'s become +1. For odd numbers, the successive periods interfere destructively, and the (±1)'s alternate. They will cancel out each other for integer values of normalized wavenumbers, but they can interfere constructively for 1/2 shifted wavenumbers. By selecting the phase-shift positions carefully, it is possible to produce a given number of large peaks, where most cosines interfere constructively. Even numbers (M) of phase shifts within a period can produce $2M$-1, while odd numbers of M can yield up to $2M$ large peaks.

We derived the effective grating coefficients (κ_{eff}) from spectral power densities. For sampled-grating κ_{eff} can be obtained by multiplication by the fill factor (a): $\kappa_{eff,S} = a\kappa$. As for the phase-modulated grating, the spectral power density depends on the x_j positions, but can be overestimated as $\frac{\left|F_P(\hat{k})\right|^2}{(N\Lambda_S)^2} \xrightarrow[\hat{k}\to 0]{} \frac{1}{N_{peak}}$, which means $\kappa_{eff,P} < \kappa/\sqrt{N_{\mathrm{peak}}}$. In practical cases ($a$=1/10, N_{peak}=7) $\kappa_{eff,P}$ can be around 3.5 times larger than $\kappa_{eff,S}$.

III. SIMULATION BASED GRATING COMPARISON

We designed a seven-peaked reflectivity comb with flat envelope and portioned a conventional DFB laser into ten periods, 120 μm (500 Λ) each, and four $\Lambda/2$-shifts were applied at calculated positions in every section. The optical modes of this phased-grating DFB are depicted with violet circles on the loss-wavelength graph [Fig. 1]. However, there is no central mode at 1550 nm with loss minimum due to the lack of the usual λ/4-shift at the device center. If it is also applied besides the 10×4 shifts, the central lowest loss mode appears (blue circles).

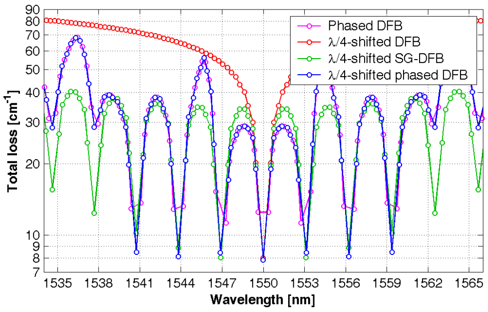

Fig. 1. Loss versus wavelength comparison of various DFB laser cavities

This phased-DFB was compared to a simple λ/4-shifted DFB, and to a sampled-grating DFB with central λ/4-shift (red and green circles). The last device also contains ten periods of 120 μm long sections and a burst length of 12 μm (10% fill factor). The grating coefficients were set as $\kappa_{SG} = 10 \cdot \kappa_{DFB}$, and $\kappa_{PG} = 3.4 \cdot \kappa_{DFB}$, where $\kappa_{DFB} = 20.8\ cm^{-1}$ to achieve same grating strength. The loss spectra of λ/4-shifted phased-DFB shows seven almost uniform peaks, others suppressed, while sampled-grating peaks possess curved envelope. The peak widths are very similar. The simulations were performed using the finite difference solution of Coupled Mode Theory [5]. The sampled-grating was handled as modulated coupling coefficient, while cumulative phase shift was used for phase-modulation, which gave the deviation from a uniform grating.

IV. FINE-SHAPING OF REFLECTION SPECTRA WITH PHASE OPTIMIZATION

The number of phase-shifts in a grating with finite number of reflection peaks can be determined using analytical equations, but their positions must be set under two criteria: desired envelope and suppression of side-peaks. We implemented a quasi-stochastic method, which starts with randomized phase-shift positions and only the best ones are stored. This process turns into proximity random optimization, where the new positions are randomly selected only in the adjacency of previous best positions.

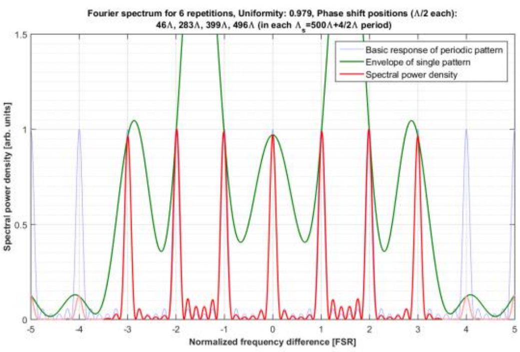

Fig. 2. 7-peaked phase-modulated grating design with flat envelope

Fig. 2. illustrates a 7-peaked example with flat-topped envelope (*M* = 4, *N* = 6, $\Lambda_S = 500\ \Lambda$): final comb together with the basic response of a periodic pattern, and the envelope resulting from phase-shifts within a single period. The peak uniformity is defined as the ratio of the smallest and largest of the peaks (>0.95). The title lists the positions of phase-shifts in the repeating sections. Only their effects are considered in our optimization method, therefore, the final physical section length must be corrected to (500+4/2) Λ.

We demonstrate that the envelope can be changed by changing the criterion in the optimization. Fig. 3. shows a convex parabolic envelope example with the depth of 0.3.

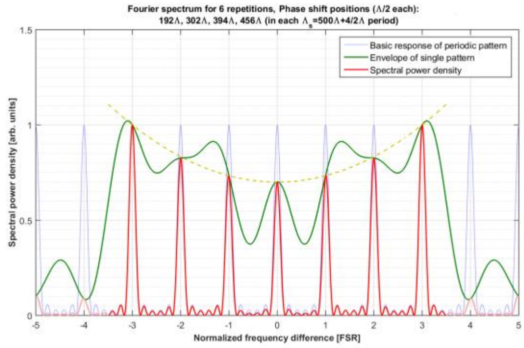

Fig. 3. 7-peaked phase-modulated grating design with curved envelope

In typical widely tunable lasers ~10 reflection peaks of DBRs are used for wavelength tuning [6,7]. For these peak numbers, good results can be achieved with discrete $\Lambda/2$ phase-shifts, and not necessary to adopt finer phase-shift schemes, which can result increased sensitivity to fabrication and optimization time (several hours compared to minutes or few tens of minutes).

V. CONCLUSION

We demonstrated that phase-modulated gratings can provide unique features, much beyond the possibilities of sampled-gratings. We derived their larger effective grating coefficient and showed that their comb-like reflectivity response can be controlled by numerical optimization of phase-modulation function. The described algorithm was successfully used for the grating design of a compact PG-DBR/Ring laser module [7].

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