Statistical Analysis of Nonlinear Harmonic Distortions in Single Drive Mach Zhender Modulators

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***Abstract*—In this preliminary investigation, a statistical approach is proposed to analyze the nonlinear harmonic distortions introduced by Mach-Zehnder modulators (MZMs) in response to arbitrarily random radio frequency (RF) excitation signals. The proposed analysis is based on a simple and accurate reformulation of the MZM transfer characteristics using the Taylor series expansion and the Binomial Theorem. Based on this reformulation, a statistical definition for the total harmonic distortion (THD) is introduced. This definition is evaluated for the conventional case of a single tone excitation. Simulation results show a promising agreement between the proposed definition and its deterministic counterpart, which is based on the Fourier analysis technique. Furthermore, Gaussian distributed RF random signals are also considered.**

***Index Terms*—Mach-Zhender modulator (MZM), nonlinear distortion, total harmonic distortion (THD).**

I. Introduction

Over the past two decades, the multi-disciplinary field of microwave photonics (MWP) has been a research topic of interest as an enabling technology for the transmission of broadband radio frequency (RF) signals over optical networks, taking the advantage of the very low propagation loss in optical fibers. In radio-over-fiber (RoF) systems, electro-optic (EO) conversion is usually accomplished via an external EO modulator such as the Mach-Zehnder modulator (MZM) owing to the numerous advantages it offers such as its low chirp characteristics. However, since it is inherently an optical phase modulator, the EO conversion via an MZM is usually susceptible to nonlinear distortions, such as the inter-modulation distortion (IMD) in multi-carrier systems, which deteriorate the end-to-end performance of the entire RoF system.

Due to its paramount importance as a key element in these systems, several studies have been reported to characterize and/or mitigate the effects of the nonlinear response of the MZM based on different models such as the truncated Taylor series model [1] and the Volterra series model [2]. A common limitation associated with these approaches is that the harmonic analysis is always based on using the Fourier coefficients of a limited number of selected harmonics at a fixed bias point, while assuming only a single tone [3] or a multi-tone [1],[4] sinusoidal excitation. For instance, in [5] a sinusoidal signal driving the MZM input at a fixed bias point was assumed. The distortions caused by the third-order and fifth-order nonlinearities were the only considered cases. However, in practice, the driving RF signal at the MZM input is a random process that cannot be modeled analytically. Instead, the statistical properties of this RF signal is the only available information. To the best of the authors' knowledge, no study has been reported to characterize the nonlinear response of MZMs using statistical methods. In this work, two different excitation signals, each with its unique and distinct statistical properties, are considered as the RF input to a typical MZM. A powerful advantage of the proposed analysis is its applicability to arbitrarily biased MZMs to predict n^{th} order distortions.

II. Proposed Analysis

The total photocurrent at the output of an intensity modulation/direct detection (IM/DD) RoF link with a single drive MZM and an ideal photodiode with a responsivity of $\Re$ is given by [6]:

$$i_{PD}(t) = \Re P_i T(t) = \Re P_i \cos^2\left(\frac{\pi}{2}\frac{v_{IN}(t)+V_B}{V_\pi}\right) \tag{1}$$

where P_i is the power of the lightwave carrier, $T(t)$ is the transfer characteristics of an ideal MZM, $v_{IN}(t)$ is the RF driving signal at its input and V_B is its DC bias voltage. The term $v_{IN}(t)+V_B$ can be reformulated as follows: $v_{rf}(t)+V_B^{'}$, where $v_{rf}(t)$ is a DC-free RF signal, $V_B^{'} = \mu_{RF}+V_B$ and μ_{RF} is the DC component of $v_{IN}(t)$. For simplicity, without loss of generality, let $P_i\Re \triangleq 1$. Using the Taylor series expansion, followed by a Binomial expansion to (1) can be expanded as follows:

$$\begin{aligned} i_{PD}(t) &= \frac{1}{2}+\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{(2n)!V_\pi^{2n}}\left(v_{rf}(t)+V_B^{'}\right)^{2n} \\ &\simeq \frac{1}{2}+\frac{1}{2}\sum_{n=0}^{N}\frac{(-1)^n\pi^{2n}}{(2n)!V_\pi^{2n}}\sum_{m=0}^{2n}\frac{(2n)!V_B^{'(2n-m)}}{m!(2n-m)!}v_{rf}^m(t) \end{aligned} \tag{2}$$

where N is the highest order term to which the Taylor series is truncated. The photocurrent in (2) can be decomposed into three components; the desired signal, a dc component and higher order terms (H.O.T) that contain in-band spectral contributions by the desired signal as well as its harmonics. This can be expressed mathematically as follows:

$$i_{PD}(t) \simeq I_{DC}+k_1 v_{rf}(t)+\sum_{m=2}^{2N}k_m v_{rf}^m(t) \tag{3}$$

A single tone input is often expressed in terms of its amplitude and frequency. In this case, the total harmonic distortion (THD), defined as the ratio of the total power of the harmonics to the power of the desired signal, is evaluated using the

Fourier coefficients of the output photocurrent. Alternatively, a sinusoidal signal can be also modeled in terms of its probability distribution function (PDF), which is given by the arcsine distribution [7]. We propose to define the THD for probabilistic signals as follows:

$$
\begin{aligned}
\mathrm{THD} \;=\; & \frac{1}{\mathrm{Var}\left(k_1 v_{rf}(t)\right)} \left\{ \sum_{m=2}^{2N} \mathrm{Var}\left(k_m v_{rf}^{m}(t)\right) \right. \\
& \left. + \sum_{i,j=2,i\neq j}^{2N} \mathrm{Cov}\left(k_i v_{rf}^{i}(t), k_j v_{rf}^{j}(t)\right) \right\} \qquad (4)
\end{aligned}
$$

where $\mathrm{Var}\,(.)$ and $\mathrm{Cov}\,(.)$ are the statistical variance and covariance operators, respectively. The definition in (4) is applied to a Gaussian distributed input voltage with a standard deviation of σ; $p(v_{rf}) = (\sqrt{2\pi\sigma^2})^{-1}\exp(-(v_{RF}^2/2\sigma^2)$, as it models a wide range of signals, including the additive white Gaussian noise (AWGN) and orthogonal frequency division multiplexed (OFDM) signals [5]. It should be highlighted that, the central moments [8] are employed to evaluate (4) for the considered distributions.

III. Results and Discussion

To validate the proposed definition in (4), numerical simulations are conducted using MATLAB. Throughout the simulations, $v_{rf}(t)$, is represented by a sample realization vector that contains 10^5 independent and identically distributed samples. Three sample realization vectors, corresponding to a variance of σ^2 = 0.01, 0.1 and 0.5, are generated for each of the considered distributions. Figure 1 compares the statistical definition proposed in (4) to evaluate the THD with its deterministic technique for the single tone case. Obviously, the results obtained form the statistical definition follows the profile of the Fourier-based definition closely with a maximum difference of 10 dB at v_{IN}/V_π = 0.05. The slight deviation of the proposed metrics just above the Fourier-based definition is attributed to the in-band spectral contributions of the H.O.Ts in (4) that are not included to the variance of the desired signal.

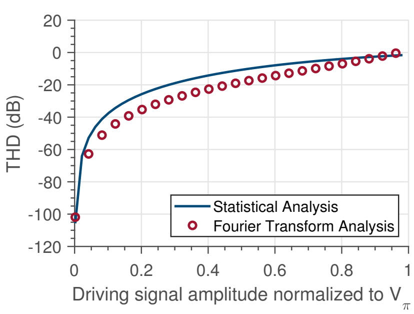

Fig. 1. The statistical and the deterministic definitions of the THD as a function of the amplitude of the RF input sinusoid at the negative quadrature bias point.

Figure 2 plots the THD versus $V_B^{'}/V_\pi$, for the two considered PDFs. Results obtained from the theory, using the central moments in [8], as well as the results obtained from direct numerical computations are shown on the same figure. Obviously, both result sets show excellent agreement for both distributions. The THD achieves its minimum at the negative quadrature bias point, and, as expected, increasing the variance, or equivalently, the amplitude, of the driving RF signal is accompanied by a corresponding increase in the THD.

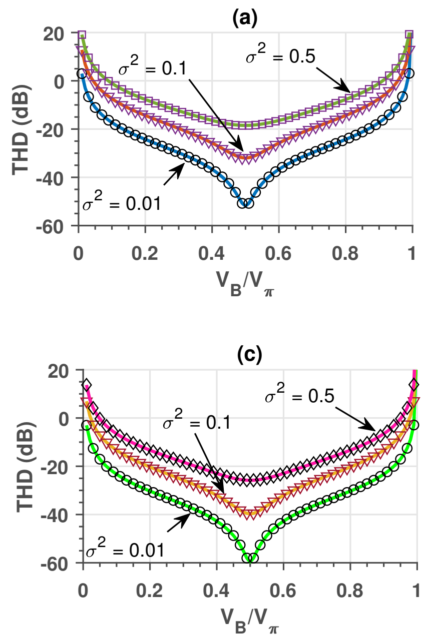

Fig. 2. The total harmonic distortion exhibited by an ideal MZM versus its bias voltage for (a): a Gaussian distributed and (b): an arcsine distributed driving signal.

IV. Conclusion

Throughout this summary, we propose a statistical definition to quantify the total harmonic distortion introduced to arbitrarily random RF signals as a result of the opto-electronic conversion via a typical MZM. Ongoing investigations are being conducted to further improve the accuracy of this preliminary investigation as well as extend its applicability to other probabilistic signals.

References

[1] V. J. Urick, M. N. Hutchinson, J. M. Singley, J. D. McKinney, and K. J. Williams, “Suppression of even-order photodiode distortions via predistortion linearization with a bias-shifted Mach-Zehnder modulator,” Opt. Express, vol. 21, no. 12, p. 14368, 2013.

[2] C. R. S. Williams, M. N. Hutchinson, T. E. Wilson, and J. M. Nichols, “Volterra Modeling of Wideband Behavior of MZM and Photodiode IMD2,” 31st Annu. Conf. IEEE Photonics Soc. IPC 2018, pp. 1–2, 2018.

[3] J. Švarný, “Bias driver of the Mach-Zehnder intensity electro-optic modulator , based on harmonic analysis,” in Advaces in Robotics, Mechatronics and Circuits, 2014.

[4] Y. Cui et al., “Intermodulation distortion suppression for intensity-modulated analog fiber-optic link incorporating optical carrier band processing,” Opt. Express, vol. 21, no. 20, p. 23433, 2013.

[5] S. Zhou et al., “Performance of optical OFDM transmission over RoF system with mach-zehnder modulator,” in Optics InfoBase Conference Papers, 2011, doi: 10.1117/12.903898.

[6] X. N. Fernando, Radio over Fiber for Wireless Communications: From Fundamentals to Advanced Topics. 2014.

[7] B. C. Arnold and R. A. Groeneveld, “Some Properties of the Arcsine Distribution,” J. Am. Stat. Assoc., vol. 75, no. 369, pp. 173–175, Mar. 1980, doi: 10.1080/01621459.1980.10477449.

[8] C. Walck, “Hand-book on STATISTICAL DISTRIBUTIONS,” Hospitals (Lond)., 1976.