# Dynamical Model of Optical Polarisation Rotation in a Charged Quantum Dot - Micropillar System 

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#### Abstract

We employ a quantum master equations approach based on a vectorial Maxwell-pseudospin model to describe the polarisation dynamics of a negatively charged quantum dot embedded in a micropillar cavity upon an ultrashort optical excitation. We demonstrate numerically a giant optical polarisation rotation ( $\sim \pm \pi / 2$ ) of a resonant circularly polarised pulse in realistic dot-cavity geometries. The model allows for optimisation of the polarisation rotation angle for realisation of spin-photon entanglement and ultrafast polarisation switching on a chip.

Index Terms-trion, cavity-dot system, polarisation rotation


## I. Introduction

Realisation of a deterministic spin-photon entanglement in semiconductor quantum dots (QDs) is one of the major goals of integrated quantum photonics. In general, there are two possible ways of realising a quantum superposition of states: through controlled rotations of either 1) the material spin qubit or 2) the photon polarisation state representing the qubit. Coupling a stationary spin qubit to an optical cavity enhances the efficiency of spin-photon interaction. Recently, a macroscopic Kerr rotation of photon polarisation ( $\sim 6^{\circ}$ ) has been observed in both the strong- [1] and weak-coupling [2] regimes in a charged QD-micropillar.

In this work, we develop a dynamical model and investigate numerically realistic micropillar cavity-dot geometries. We show that a resonant coherent interaction of an ultrashort circularly polarised pulse with the trion transition results in a giant rotation angle of $\pi$ in the weak-coupling regime. Large rotation angles are highly desirable for fabrication of phase gates for optical quantum computing and enable using charged QD-cavities as ultrafast polarisation switches on a chip.

## II. Theoretical Model

We consider a negatively charged InGaAs QD embedded in a AlAs/AGaAs micropillar cavity Fig. 1(a). An equivalent four-level scheme of the ground trion singlet transition and the relevant spin relaxation processes is shown in Fig. 1(b). The dynamical evolution of an open 4-level quantum system under a time-dependent perturbation is described by a master
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pseudospin equation for the real state vector, $S_{j}$ [3], [4]:

$$
\frac{\partial S_{j}}{\partial t}=\left\{\begin{array}{lc}
f_{j k l} \Gamma_{k} S_{l}+\frac{1}{2} \operatorname{Tr}\left(\hat{\sigma} \cdot \hat{\lambda}_{j}\right)-\frac{1}{T_{j}}\left(S_{j}-S_{j e}\right), \quad j=1,2, . ., 12  \tag{1}\\
f_{j k l} \Gamma_{k} S_{l}+\frac{1}{2} \operatorname{Tr}\left(\hat{\sigma} \cdot \hat{\lambda}_{j}\right), & j=13,14,15,
\end{array}\right.
$$

where $\Gamma_{k}$ - torque vector, $\widehat{\lambda}_{j}$-generators of $S U(4)$ Lie group algebra, $f$ - fully-antisymmetric tensor of the structure constants; $\hat{\sigma}$ - population relaxation; $S_{j e}$ - equilibrium coherence vector, and $T_{j}$ - phenomenological decay times towards $S_{j e}$.

We solve self-consistently the quantum evolution equations Eqs. (1) and the vector Maxwell equations for a resonant circularly polarised optical pulse interacting with an ensemble of four-level systems (charged QDs) with density $N_{d}$ :

$$
\begin{align*}
& \frac{\partial H_{x}(z, t)}{\partial t}=\frac{1}{\mu} \frac{\partial E_{y}(z, t)}{\partial z} \\
& \frac{\partial H_{y}(z, t)}{\partial t}=-\frac{1}{\mu} \frac{\partial E_{x}(z, t)}{\partial z}  \tag{2}\\
& \frac{\partial E_{x}(z, t)}{\partial t}=-\frac{1}{\varepsilon} \frac{\partial H_{y}(z, t)}{\partial z}-\frac{1}{\varepsilon} \frac{\partial P_{x}(z, t)}{\partial t} \\
& \frac{\partial E_{y}(z, t)}{\partial t}=\frac{1}{\varepsilon} \frac{\partial H_{x}(z, t)}{\partial z}-\frac{1}{\varepsilon} \frac{\partial P_{y}(z, t)}{\partial t}
\end{align*}
$$

where the polarisation components are given by: $P_{x}=$ $-\wp N_{d} S_{1} ; P_{y}=-\wp N_{d} S_{7} . \wp=\langle 1| e \mathbf{r}|2\rangle=\langle 3| e \mathbf{r}|4\rangle$ is the dipole matrix element. The optical rotation angle of a transmitted through the cavity pulse is calculated from the phase shift induced by the resonant absorption of the QD layer with thickness $l=z_{2}-z_{1}$. The resonant phase shift is given by $\Delta \varphi_{x, y}=\beta_{x, y} l$, where $\beta_{x, y}=\frac{1}{l} \arctan \left(\frac{\operatorname{Im}\left(z_{x, y}\right)}{\operatorname{Re}\left(z_{x, y}\right)}\right)$ and $z_{x, y}=\frac{E_{x, y}\left(z_{2}, \omega\right)}{E_{x, y}\left(z_{1}, \omega\right)}=e^{i k_{c x, y} l}$ is the complex propagation factor, given by Fourier transform of the time traces of the $E_{x}$ and $E_{y}$ sampled at the end points of the QD layer (Fig. 2(a)).

| $\begin{gathered} E_{\text {trion }} \\ \text { [eV] } \\ \hline \end{gathered}$ | $\begin{gathered} \lambda_{0} \\ {[\mathbf{n m}]} \\ \hline \end{gathered}$ | $\begin{gathered} T_{p} \\ \text { [fs] } \\ \hline \end{gathered}$ | $\begin{gathered} E_{0} \\ {\left[\mathrm{Vm}^{-1}\right]} \\ \hline \end{gathered}$ | $\begin{gathered} \wp \\ {[\mathrm{e} . \mathrm{nm}]} \end{gathered}$ | $\begin{gathered} N_{d} \\ {\left[m^{-3}\right]} \end{gathered}$ | Relaxation times [ps] |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\Gamma$ | $\gamma_{1}$ | $\gamma_{2}$ | $\gamma_{3}$ | $\Gamma_{\tau}$ |
| 1.388 | 894 | 100 | $1.44 \times 10^{8}$ | 0.57 | $3.18 \times 10^{24}$ | 820 [2] | 500 [5] | 450 [7] | 170 [6] | 340 |

## III. Numerical method and results

Eqs. (1-2) are solved directly in the time domain by the Finite-Difference Time-Domain (FDTD) method. The resonant Gaussian circularly polarised pulse, is given by:

$$
\sigma^{ \pm}\left\{\begin{array}{l}
E_{x}(z=L, t)=E_{0} \exp \left[-\left(t-t_{0}\right)^{2} / t_{d}^{2}\right] \cos \left(\omega_{o} t\right)  \tag{3}\\
E_{y}(z=L, t)= \pm E_{0} \exp \left[-\left(t-t_{0}\right)^{2} / t_{d}^{2}\right] \sin \left(\omega_{o} t\right)
\end{array}\right.
$$



Fig. 1. (a) Sketch of a charged QD-micropillar structure; $\kappa$ - cavity loss, $\Gamma_{\tau}-$ trion decay rate to non-cavity modes. The micropillar cavity is pumped from the top (through an optical fibre) by an ultrashort circularly polarised pulse; (b) Energy-level diagram of the negative trion ground singlet transition driven resonantly by either left- $\left(\sigma^{-}\right)$or right $\left(\sigma^{+}\right)$circularly polarised light. Electron $\left(\gamma_{1}\right)$ / hole $\left(\gamma_{3}\right)$ : spin flip relaxation rates; $\Gamma$ : spontaneous emission rate; $\gamma_{2}, \Gamma_{\tau}$ : spin decoherence rates. (c) Refractive-index profile (black line) of the micropillar with embedded QD [2], and a snapshot of the E-field components (yellow and cyan curves) and level populations (red, blue, green and magenta) at $\mathrm{t}=3.33 \mathrm{ps}$. (c) Zoom-in on (b) of the QD layer.


Fig. 2. (a) Refractive index profile of the micropillar from [2]; inset: zoom-in of the region around the QD layer; $z_{1}$ and $z_{2}$ denote the left and right end points of the QD layer; (b) Time evolution of the E-field components of a $\sigma^{-}$pulse and level populations at $z_{2}$ for a spin-down initial state; $\rho_{22}$ (blue curve) proportional to experimentally detected TRPL; (c) Phase shift (in degrees) of the $E_{x}$ and $E_{y}$ pulse components of a $\sigma^{-}$- pulse. Blue curve: stationary solution of the density-matrix equations for a two-level system (no cavity), (d) Phase shift of the E-field components of a linearly x-polarised pulse.
where $\pm$ corresponds to right(left) circularly polarised pulse with amplitude $E_{0}$, or $E_{y}=0$ for $x$ - linearly polarised pulse. The model parameters are summarised in Table I.

Consider initial spin-down population prepared entirely in ground level $|1\rangle$ e.g. by e.g. optical pumping [8]. The temporal dynamics of the E-field components of a $\sigma^{-}$pulse and populations of all levels is calculated at the end points, $z_{1}$ and $z_{2}$, of the QD layer (see Fig.2(a,b)). We numerically demonstrate a giant phase shift of $\pm \pi / 2$, induced by the single spin confined in the QD in a range of realistic micropillar cavities (Fig.2(c)). We calculate the cavity transmission spectrum and the cavity loss, $\kappa=1.25 \mathrm{ps}^{-1} \gg \Omega_{R} \sim 125 \mathrm{ps}^{-1}$ - weak coupling regime. By contrast, when the trion transition is driven by a resonant $x$-linearly polarised pulse, we show that the orthogonal (y) E-field pulse component (initially set to zero) is building up in time, thereby leading to a polarisation rotation. The time delay in the $E_{y}$ amplitude build-up results in a red shift in the phase shift spectrum with respect to the one for the $E_{x}$ (Fig.2(d)). This displacement effectively leads to destructive interference and significant reduction of the optical rotation angle. In summary, we demonstrate numerically a
giant $\sim \pm \pi / 2$ optical rotation for a circularly polarised ultrashort pulse, which exceeds by an order of magnitude the Kerr polarisation rotation angles obtained by cw linearly polarised excitations. Our method allows to design and test cavity-dot structures with maximised photon polarisation rotation angles and thus prepare high-fidelity photon qubit states.

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