Carrier Recombination in Semiconductor Lasers: Beyond the ABC

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ABC's of Semiconductor Lasers

Classical Parametrization of Loss Current $J_{\text{loss}}$:

$$J_{\text{loss}} = A N + B N^2 + C N^3 + J_{\text{rest}}$$

- **Defect-recombination**
- **Spontaneous emission**
- **Auger recombination**
- **Non-capture, escape**
- Usually negligible in high quality crystal growth
- Usually dominant
- Absent in optically pumped devices

Problems with $A$, $B$, $C$ - Parametrization:

- Parameters only very roughly known and only for special cases;
  - Depend on well- and barrier-materials, layer widths, temperatures, densities...
- Simple density-dependence far from reality
Problems with B, C:  \( J_{\text{spont}} = B N^2 \) ?  
\( J_{\text{aug}} = C N^3 \) ?

**Low Density:**  \( E_g - \mu \gg 1/\beta = k_B T \)
Maxwell-Boltzmann Distributions:  \( \implies \)

\[ J_{\text{spont}}(\text{low density}) \approx B N^2 \]
\[ J_{\text{aug}}(\text{low density}) \approx C N^3 \]

**High Density:**  \( \mu - E_g \gg k_B T \)
Fermi Distributions:  \( f \approx \begin{cases} 0 & \text{if } k > k_F \\ 1 & \text{if } k < k_F \end{cases} \)

\[ J_{\text{spont}}(\text{high density}) \approx B N, \]
\[ J_{\text{aug}}(\text{high density}) \approx C N^x, \; x < 3 \]
Gain:

Semiconductor Bloch equations (SBE):

\[
\frac{d}{dt} P_{k}^{ji} = \frac{1}{i\hbar} \left\{ \sum_{i',j'} \left[ \mathcal{E}_{k}^{h,j',k} \delta_{i',i} + \mathcal{E}_{k}^{e,i',k} \delta_{j,j'} \right] P_{k}^{j'i'} + \left[ 1 - f_{k}^{e,i} - f_{k}^{h,j} \right] \mathcal{U}_{i,j,k} \right\} + \frac{d}{dt} P_{k}^{ji} \bigg|_{\text{corr}}
\]

\[
\mathcal{E}_{k}^{e,i',k} = \varepsilon_{k}^{e,i} \delta_{i',i} - \sum_{i'',q} V_{k,q}^{i'i''} f_{q}^{e,i''} \quad \mathcal{U}_{i,j,k} = -\mu_{ij,k} E(t) - \sum_{i',j',q} V_{k,q}^{i'j'} f_{q}^{j'i'}
\]

Quantum-Boltzmann scattering in 2. Born-Markov approximation to determine dephasing of \( P \), lineshape of \( \alpha(\omega) \):

\[
\frac{\hbar}{\pi} \frac{d}{dt} P_{k}^{ji} \bigg|_{ee} = \sum_{n,k',q} 2 \left| \tilde{V}_{q}^{inni} \right|^{2} D \left( \varepsilon_{k' + q}^{e,i} - \varepsilon_{k}^{e,i} - \varepsilon_{k}^{n} + \varepsilon_{k' - q}^{n} \right) \times
\]

\[
\left[ f_{k}^{e,i} f_{k'}^{n} (1 - f_{k' - q}^{n}) + (1 - f_{k}^{e,i}) (1 - f_{k'}^{n}) f_{k' - q}^{n} \right] P_{k + q}^{ji} + \ldots
\]
Theory

Gain:

With explicit treatment of scattering:
- correct amplitudes, spectral positions, shifts
- no unphysical absorption
- correct density dependence for SE and gain

Without explicit treatment of scattering but lineshape functions:
- wrong amplitudes, spectral positions, shifts
- unphysical absorption
- drastically wrong density dependence for gain and SE

(From J. Hader, et al., IEEE J. Sel. Topics Quantum Electron. 9, 688 (2003))
Spontaneous Emission; KMS vs. SLE:

\[ J_{SE} = eR_{SE} = e \int d\omega S(\omega) \]

Kubo Martin Schwinger Relation (KMS) between absorption/gain, \( \alpha(\omega) \), and SE, \( S(\omega) \):

\[ S(\omega) = -\frac{1}{\hbar} \left( \frac{\epsilon_b\omega}{\pi c} \right)^2 \alpha(\omega) \left[ e^{\frac{\hbar\omega - \mu}{k_B T}} - 1 \right]^{-1} \]

Semiconductor Luminescence Equations (SLE):

- Equations of motion for photon assisted polarizations: \( <b^+v^+c> \)
- Similar to SBE, i.e. equations of motion for polarizations: \( <v^+c> \), \( <c^+v> \)
- Scattering in 2. Born-Markov approximation
Theory

Spontaneous Emission; KMS vs. SLE:

**KMS:**
- numerically very simple
- ok for low density lineshapes
- fails close to transparency
- overestimates low density SE
- some tens of percents wrong in the gain regime

**SLE:**
- more complex numerically
- ok for high density lineshapes
- accurate for transparency density
Auger Recombination:

Quantum-Boltzmann scattering in 2. Born-Markov approximation to determine Auger transitions

\[
\frac{d f_{i,s}^{i',s'}}{dt} = \frac{2\pi}{\hbar} \sum_{k',q,s'} \Re \left\{ \sum_{j_1,j_2,j_3} \left( |\tilde{V}_{q}^{i,j_3,i_1,j_2} - s' - s\rangle - |\tilde{V}_{q}^{i,j_3,i_1,j_2} - s' - s\rangle |k' - q + k| \right)^2 \right\} \times \\
\mathcal{D} \left( -\varepsilon_k^{i,s} - \varepsilon_{|k'-q|}^{j_1,s'} - \varepsilon_{|q-k|}^{j_2,-s} + \varepsilon_{k'}^{j_3,s'} \right) \times \\
\frac{1}{2} \left[ f_{k'}^{j_3,s'} \left( 1 - f_{|q-k|}^{j_2,-s} \right) \left( 1 - f_{|k'-q|}^{j_1,s'} \right) \left( 1 - f_k^{i,s} \right) - \right. \\
\left. \left( 1 - f_{k'}^{j_3,s'} \right) \frac{f_{|q-k|}^{j_2,-s} f_{|k'-q|}^{j_1,s'} f_k^{i,s}}{f_{k'}^{j_3,s'} f_{q-k}^{j_2,-s} f_{k'-q}^{j_1,s'} f_k^{i,s}} \right] + \ldots
\]
Theoretical Procedure:

- calculate gain for various densities
- search for density that overcomes intrinsic losses (mirror losses) = threshold density
- calculate spontaneous emission and Auger recombination for this density

*put on top of experimental result without adjustment*

Theory-Experiment Comparison

![Graph showing the comparison between theoretical and experimental results for 6nm wide GaInNAs well lasing at 1300nm. The graph plots threshold current against temperature.]
Theory-Experiment Comparison

4 x 2.5nm wide InGaAsP-well, lasing at 1500nm

8 x 4.4nm wide InGaAsP wells, lasing at 1300nm

How good are the ABC's?:

- Error of more than two already at transparency for B- and C-laws.
How good are the ABC’s?:

- $J_{\text{spont}}$ increases only linear with $N$ at high densities
- $J_{\text{aug}}$ increases far less than cubic with $N$; sometimes even less than quadratic
Closed-Loop Laser Design

Predicting Input-Output Characteristics Using Basic Structural Information


Experimental Input:
- nominal structural parameters (layer widths, material compositions, device length, $L$,
  mirror reflectivities, $R_1$, $R_2$
- outcoupling loss, $\alpha_{out} = 1/(2L) \ln[1/(R_1 R_2)]$
- internal loss $\alpha_{int}$
- threshold loss, $\alpha_{thr} = \alpha_{int} + \alpha_{out}$
- low excitation PL

Step 1:
- calculate PL using fit parameter free SLE's;
  compare to measured PL
  inhomogenous broadening and actual structural compositions

Step 2:
- calculate gain using fit parameter free SBE's and apply inhomogeneous broadening;
  look up density for which gain compensates $\alpha_{thr}$
  threshold density, $N_{thr}$

four 6nm wide InGaAsP/InP wells
Step 3:

- use fit parameter free SLE's and Auger model to calculate spontaneous emission- and Auger-losses at threshold, $J_{se}(N_{th})$, $J_{aug}(N_{th})$.

threshold current, $J_{SE}(N_{th})+J_{aug}(N_{th})$
Step 4, Comparison to Experiment:

Assumptions:
- slope efficiency = $\alpha_{\text{out}} / \alpha_{\text{thr}}$
- internal efficiency = 100%
- homogeneous mode under pumped area
- No adjustments of any parameters.
- No free parameters.
- True predictions for threshold and temperature dependence.

NOTE:
When using adjustable parameters like an Auger-constant, C, and its temperature dependence, a reasonable FIT to the threshold and its temperature dependence can always be obtained.
Summary

Spontaneous Emission:

- B$_2$N$_2$-assumption leads to an error of several orders of magnitude even if low-density B is known
- above threshold N$_2$-assumption completely breaks down
- here, only linear increase with density due to phase space filling
- Numerically expensive SLE’s have to be used especially for densities near transparency

Auger Recombination:

- C$_3$N$_3$-assumption leads to an error of up to one order of magnitude even if low-density C is known
- measured and/or calculated literature values for C vary by 1-2 orders of magnitude for similar systems
- C strongly temperature- and density dependent
- N$_{\text{thr}}$ 25% wrong Auger-current wrong by factor 2

Shortcomings of Simpler Approaches

Dephasing Time Approximation:

threshold density overestimated by about factor of 2

\[\text{up to one order of magnitude error in loss-currents}\]
Shortcomings of Simpler Approaches

Bulk Approximation for Barrier States:

- subband approximation:
  - similar density of states as bulk
  - *seems* to be good for periodic MQW systems
  - neglects coupling between well-unit-cells
  - neglects formation of subbands and mixing of wavefunction-character

- bulk approximation:
  - good for total barrier widths of more than about ten excitonic Bohr radii
Shortcomings of Simpler Approaches

Bulk Approximation for Barrier States:

- unphysical resonances in width dependence
- wrong by factor of about 2


7 wells, 4nm barriers

C = 2.4