Domain Decomposition Method for Electromagnetic Scattering Problems: Application to EUV Lithography

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Photolithography

- Imprint the integrated circuits to a semi-conductor chip
- Transfer mask pattern by optical projection

Left: State-of-the-art tools, wavelengths of 250 and 190 nm. Minimum feature sizes down to 65nm.

Right: Extreme Ultraviolet (EUV) radiation system, 13nm wavelengths. Dramatically reduced resist feature sizes.

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EUV Mask

- Multilayer coating serves as a mirror.
- Leads to huge computational domains.

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Domain Decomposition

Solve scattering problem in upper domain with incoming waves $E_{in}$ and $E_n$

Compute $E_{out}$

Solve scattering problem in lower domain with incoming wave $E_{out}$

Compute reflected wave $E_{n+1}$

See also: Despres ('97), Toselli ('98), Colino & Joly ('00), Gander ('02)

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Scattering Problems:

- Finite elements with transparent boundary condition in structure blocks and for *perturbed* layer blocks.
- Fourier-mode expansion in *non-perturbed* layer block (semi-analytic solution)

Coupling Problems:

- FEM – FEM Coupling (with transparent boundary conditions!)
- FEM – Fourier Mode Coupling
- Fourier Mode – FEM Coupling
DD: Maxwell’s Equations

Periodic and $z$–invariant Media:

$$
\varepsilon(x + a, y) = \varepsilon(x, y), \quad \mu(x + a, y) = \mu(x, y)
$$

Bloch periodic incoming field:

$$
E_{\text{inc}}(x + a, y, z) = E_{\text{inc}}(x, y) e^{ik_x a} e^{ik_z z}
$$

Maxwell’s equation:

$$
\text{curl}_{k_z} \mu^{-1} \text{curl}_{k_z} E - \omega^2 \varepsilon E = 0
$$

with $\text{curl}_{k_z} = \text{curl}, \partial_z \leftrightarrow ik_z$

Bloch Periodicity:

$$
E(x + a, y) = E(x, y) e^{ik_x a}
$$
Fourier Modes Expansion

Most simple situation: $\mu$ and $\varepsilon$ are constant

Fourier expansion of $\mathbf{E}_{\text{sc}}$:

$$
\mathbf{E}_{\text{sc}}(x, y) = \sum_n e^{i(n2\pi/a+k_x)x} \left( \bar{e}_{n,+} e^{ik_{y,n}y} + \bar{e}_{n,-} e^{-ik_{y,n}y} \right)
$$

with $k_{y,n} = \sqrt{k_0^2 - (n2\pi/a + k_x)^2 - k_z^2}$

Radiation boundary condition: $\bar{e}_{n,-} = 0$

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Anomalous Modes

- $\text{Real}(k_y) > 0$ (Radiating mode, Sommerfeld like boundary condition)

- $\text{Imag}(k_y) > 0$ (Evanescent mode)

- $k_y = 0$ (Anomalous mode)
Fourier Coupling

Fourier expansion of incoming field and reflected field:

\[
E_{\text{inc}}(x, y) = \sum_n e^{i(n2\pi/a+k_x)x} \vec{e}_{n,\text{inc}} e^{ik_y,ny}
\]

\[
E_{\text{out}}(x, y) = \sum_n e^{i(n2\pi/a+k_x)x} \vec{e}_{n,\text{out}} e^{-ik_y,ny}
\]

- \(|k_{y,n}| \geq \text{threshold} : \) Dirichlet Coupling
  \[
  \vec{e}_{n,\text{inc}} \times \vec{n} \mapsto (\vec{e}_{n,\text{out}} \times \vec{n}, ik_y, n \times \vec{e}_{n,\text{out}} \times \vec{n})
  \]
  No evaluation of \(\text{curl} E_{\text{inc}} \times \vec{n}\).

- \(|k_{y,n}| < \text{threshold} : \) Mixed Dirichlet–Neumann Coupling
  Splitting into two polarization directions.
  Use weak evaluation of \(\text{curl} E_{\text{inc}} \times \vec{n}\).
Finite Element Problem

\[ \Omega_+ = [0, a] \times [0, \infty) \]
\[ \Omega = [0, a] \times [-h, 0] \]
\[ \Omega_- = [0, a] \times (-\infty, -h] \]

Split field:

\[ E = \begin{pmatrix} E_\perp \\ E_z \end{pmatrix} \]

with

\[ E_\perp \in H_B (\text{curl}) := \{ u \in H (\text{curl}) \mid u(x + a, y) = e^{ik_x a} u(x, y) \} \]
\[ E_z \in H^1_B := \{ u \in H^1 \mid u(x + a, y) = e^{ik_x a} u(x, y) \} \]

See also: Elschner et al. (1999) Theory & \((E_z, H_z)\)-formulation
Bloch Periodic Finite Elements

Discretization of $\mathbf{H}_\perp$ with Nedelec elements,
$H_z$ with Lagrange elements.
Coupled Exterior–Interior System

\[ \Omega_+ = [0, a] \times [0, \infty) \]

**Interior Problem**

\[ \text{curl}_k \mu^{-1} \text{curl}_k E - \omega^2 \varepsilon E = 0 \]

**Exterior Problem**

\[ \text{curl}_k \mu^{-1} \text{curl}_k E_{sc} - \omega^2 \varepsilon_+ E_{sc} = 0 \]

**Matching condition at** \( y = 0 \)

\[ E \times \vec{n} = (E_{sc} + E_{inc}) \times \vec{n} \]

\[ \mu^{-1} \text{curl} E = (\mu_+^{-1} E_{sc} + \mu_+^{-1} E_{inc}) \times \vec{n} \]
Perfectly Matched Layer

**Idea:** Complex continuation \((y \rightarrow (1 + i\sigma)y)\) of \(E_{sc}(x, y)\)

\[
E_{PML}(x, y) = \sum_n \bar{e}_n e^{i k_y n (1 + i\sigma)y} e^{i(n2\pi/a + k_x)x}
\]

Exponential decaying field:

\[
|E_{PML}(x, y)| \sim e^{-\kappa y}
\]

with \(\kappa = \min_n \{\Re k_y n, \Im k_y n\}\).

**Truncate exterior domain:** \(\Omega_\rho = [0, a] \times [0, \rho]\)

\(\kappa = 0\) in the presence of anomalous modes!
PML for anomalous modes

The (near) anomalous modes are slowly varying in \( y \).

**Idea:** Use very, very large PML layer but very few discretization points.

#### Heuristical choice of \( h_{j+1} \):

Determine \( k_{y,\text{max}} \) such that

\[
e^{-i k_{y,\text{max}} \rho_j} \leq \text{threshold}, \quad \rho_j = \sum_{j' \leq j} h_{j'}
\]

Waves with \( \lambda_{\text{min}} \leq 2\pi/k_{y,\text{max}} \) are sufficiently damped

\[
\rightarrow h_j = \max\{h_1, \lambda_{\text{min}}/N\}
\]
Total reflection

Error at wood anomaly: 4.78e-3, 1.1e-3, 2.9e-4, 7.2e-5
(uniform interior grid refinement)
Real world example

Reflection off Dense Line, 40 MoSi layers, $\alpha = 4$deg, $\theta = 30$deg, P-Pol.

- 22sec 94sec 587sec
- Zero order
- First order
- Seventh order
- Eighth order

Reflection off Dense Line, 40 MoSi layers, $\alpha = 4$deg, $\theta = 30$deg, P-Pol., Domain Decomp.

- 32sec 78sec 316sec 1330sec
- Zero order
- First order
- Seventh order
- Eighth order
Outlook

- 3D Photolithography mask. All building blocks for domain decomposition available in JCMharmony (by JCMwave)

- Isolated Structures, e.g. contact holes. Finite Element solver available. Coupling of domains requires exterior field evaluation (pole condition).

Talk by Frank Schmidt et al.:

*Benchmark of FEM, waveguide, and FDTD algorithms for rigorous mask simulation*, Photomask Technology 2005, Monterey, 4 October 2005