Energy Band Calculations for Dynamic Gain Models in Semiconductor Quantum Well Lasers

P.J. Bream, S. Sujecki and E.C. Larkins

School of Electrical and Electronic Engineering
University of Nottingham; Nottingham NG7 2RD; UK

Email: eexpjb1@nottingham.ac.uk
Presentation Outline

- Fourier series representation of valence band asymmetries
- Carrier energy density of states
- Gain spectra
- Dynamic gain model
Quantum well valence band structure – Fourier expansion

- Decoupled 4-band $k.p$ method for <100> and <110> crystal directions
- 1st order Fourier expansion fulfills symmetry and continuity requirements
- Numerically efficient

$$k(\theta) = \frac{k_{<110>} + k_{<100>}}{2} - \frac{k_{<110>} - k_{<100>}}{2} \cos(4\theta)$$
Quantum well valence band structure – local maxima

- Cannot use the Fourier expansion method in this region near local maxima
- 8nm GaAs/Al_{0.2}Ga_{0.8}As QW

\[ a = k_{<110>} (E_{<110>_{max}}); b = k_{<100>} (E_{<100>_{max}}); c = k_{<100>}^- (E_{<110>_{max}}); d = k_{<100>}^+ (E_{<110>_{max}}) \]
Quantum well valence band structure – local maxima

- Series of parabolic approximations made to describe energy contours near local valence band maxima
Subband energy density of states

- Non-zero density of states near local maxima
- Location of peak shifted
- Energy density of states up to 10% larger than for circularly symmetric method
Optical energy density of states – carrier energy spread

8nm In$_{0.2}$Ga$_{0.8}$As/Al$_{0.2}$Ga$_{0.8}$As QW

\[ E_{\text{photon}} = 1.32\text{eV} \]

\[ \Delta E_{\text{carrier}} \approx 0.5\text{meV} \]

8nm GaAs/Al$_{0.2}$Ga$_{0.8}$As QW

\[ E_{\text{photon}} = 1.525\text{eV} \]

\[ \Delta E_{\text{carrier}} \approx 3.3\text{meV} \]
Gain/absorption

- Carrier energy spread affects matrix element and occupational probability
- Gain calculation requires sum over this range for each photon energy

$$g(E_{ph}) \propto \sum_{cb, vb} \int \int |M_T(E_e, E_h)|^2 \rho_{opt}(E_e, E_h) \left[ f_c(E_e) - f_v(E_h) \right] dE_e dE_h$$
Dynamic QW gain model

- States at same energy assumed to have equal occupational probability
  - ultrafast momentum relaxation at given energy
  - implicitly assumed for relaxation rate approximation
  - allows $\int d^2k$ to be replaced by $\int dE$

- Individual intrasubband, intersubband and interband relaxation processes treated separately

- Carrier-carrier scattering modelled using relaxation rate approximation

- Carrier-phonon scattering modelled using phonon emission/absorption
  - Carrier kinetic energy threshold for phonon emission

- Gain and spontaneous emission derived from band structure model
  - linewidth enhancement and bandgap renormalisation omitted for clarity
Dynamic QW gain model – spectral hole burning (case 1)

- 8nm In$_{0.2}$Ga$_{0.8}$As/Al$_{0.2}$Ga$_{0.8}$As QW
- thermal equilibrium carrier concentration of 2.5x10$^{18}$cm$^{-3}$ at t=0$^-$
- Excited by 15fs gaussian pulse, 1.35eV low energy source (25$\mu$J average)

Spectral hole in electron occupational probability distribution

Circularly symmetric band structure method

Fourier expansion band structure method
Dynamic QW gain model – spectral hole burning (case 1)

Spectral hole in hole occupational probability distribution

Circularly symmetric band structure method

Fourier expansion band structure method
Dynamic QW gain model – spectral hole burning (case 1)

Spectral hole in gain spectrum

Circularly symmetric band structure method
- Affects linewidth enhancement

Fourier expansion band structure method
**Dynamic QW gain model** – optical pumping (case 2)

- 8nm In$_{0.2}$Ga$_{0.8}$As/Al$_{0.2}$Ga$_{0.8}$As QW
- thermal equilibrium intrinsic carrier concentration at $t=0^-$
- 1MW/cm$^2$ CW pump ($E_{ph}=1.35$eV) introduced at $t=0^+$

**Electron probability distribution** – Fourier expansion band structure method

- **a** – photon absorption from HH1 to CB1
- **b** – photon absorption from HH2 to CB1
- **c,d** – LO phonon emission replicas
- **e,f** – LO phonon absorption replicas
- **g** – electrons due to carrier-carrier scattering
**Dynamic QW gain model** – phonon model

- Same QW and starting conditions
- Same optical pump but switched off after $\Delta t=100\text{fs}$
- Fourier expansion band structure method

**Electron probability perturbation (phonon relaxation only)**

**Phonon emission/absorption**

**Relaxation rate approximation**
Summary

- Numerically efficient valence band model
  - Retains important asymmetrical features of full $k.p$ model
  - Affects density of states and dynamic gain

- Dynamic gain model
  - Intrasubband, intersubband, interband processes separate
  - Carrier-phonon scattering modelled using emission/absorption
IS THE RELAXATION RATE APPROXIMATION APPROPRIATE FOR CARRIER-CARRIER SCATTERING?

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