Hierarchical models for multi-longitudinal mode semiconductor lasers

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Multi-longitudinal mode instabilities in semiconductor lasers (colloquially referred to as kinks) can drastically affect device performance and yield. Simulations that accurately predict multi-longitudinal mode instabilities use spatio-temporal partial differential equations, which can add unnecessary computational overhead. Simpler time resolved rate equations are much faster by comparison but cannot indicate potential multi-longitudinal instabilities. A solution is to develop a hierarchy of equations, ranging from a full spatio-temporal model to a simple single mode model, which are self-consistent across all levels.

The longitudinally resolved semiconductor laser rate equations are

$$\frac{n_b}{c}\frac{\partial E^\pm}{\partial t} + \frac{\partial E^\pm}{\partial z} = \frac{i}{2k_0}\frac{\partial^2 E^\pm}{\partial x^2} + \frac{\Gamma a}{2}\{(N - N_0)GD - \imath\alpha N\}E^\pm - \alpha_{int}E^\pm,$$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x}D_N\frac{\partial N}{\partial x}\frac{J(z)}{edwL} - \frac{1}{\tau}N - \frac{\epsilon_0 c}{2\epsilon_b\hbar\omega_c}a(N - N_0)(|E^+|^2 + |E^-|^2),$$

$$GD = 1 + \frac{\hbar^2}{\gamma^2}\frac{c^2}{n_b^2}\frac{\partial^2}{\partial z^2},$$

with boundary conditions

$$E^+(z = 0) = \sqrt{R_1}\,E^-(z = 0), E^-(z = L) = \sqrt{R_2}\,E^+(z = L).$$

Symbols have the meaning attributed in [1].

The self consistent coupled mode model derived from the PDE model above is [2]

$$\frac{n_b}{c}\frac{\partial E_n}{\partial t} = \frac{i}{2k_0}\frac{\partial^2}{\partial x^2}E_n - \alpha_{int}E_n + \frac{\Gamma a}{2}\sum_m GD_m N_{n-m}E_m + \frac{\Gamma a}{2}GD_n N_{tr}E_n - i\frac{\Gamma a}{2}\alpha\sum_m N_{n-m}E_m,$$

$$\begin{aligned}\frac{\partial N_n}{\partial t} &= i\omega_n N_n + \frac{\partial}{\partial x}D_N\frac{\partial}{\partial x}N_n + \frac{J}{edwL}\delta_n - \frac{1}{\tau}N_n \\ &+ \frac{\epsilon c}{\epsilon_b\hbar\omega_0}aN_{tr}\left[\sum_m \eta^{0,n}_{n+m,m}E_{n+m}E^*_m\right] - \frac{\epsilon_0 c}{\epsilon_b\hbar\omega_0}a\left[\sum_{l,m}\eta^{n+m-l,n}_{l,m}N_{n+m-l}E_l E^*_m\right],\end{aligned}$$

$$GD = 1 + \frac{\hbar^2}{\gamma^2}\frac{c^2}{n_b^2}\left[\frac{1}{4}\frac{(\ln R)^2}{L^2} + \frac{\ln R}{L}\frac{\partial}{\partial z} + \frac{\partial^2}{\partial z^2}\right],$$

$$\eta^{k,n}_{l,m} = v^2\frac{R^{-1} - 1}{iLk_{k+l-m-n} - \ln R} + \frac{1}{v^2}\frac{1 - R}{iLk_{k+l-m-n} - \ln R} + \frac{1}{v^2}\frac{1 - R}{iLk_{k-l+m-n} - \ln R} + v^2\frac{1 - R^{-1}}{iLk_{k-l+m-n} - \ln R}$$

All of the parameters are directly related to the parameters in the PDE model.

Truncated coupled mode models can be numerically integrated quickly compared to the PDE model when the laser is operating in a single mode regime. Single mode operation is occurs when the carrier grating terms ($N_1, N_2, \ldots$) are zero. When energy flow between modes is critical the carrier grating terms become non-zero (Fig. 1). This provides an monitor of when multi-mode dynamics becomes important, even if energy is predominantly in a single mode.

The kink instability in fabry-perot lasers is used as a test for the hierarchical model approach. Figure 2 shows the kink instability calculated with the PDE model and a 5 mode model using same parameters. The 5 mode model accurately predicts the current regime in which the kink occurs and where further investigation with the PDE model is needed to understand the dynamics.

[1] J. K. White et al., “Multi longitudinal mode dynamics in a semiconductor laser subject to optical injection”, *IEEE J. Quant. Electron.*, vol. 34, pp. 1469-1473 (1998).

[2] J. K. White, “Communication with chaotic semiconductor lasers”, Ph.D Dissertation, The University of Arizona, pp. 50-62 (1999).

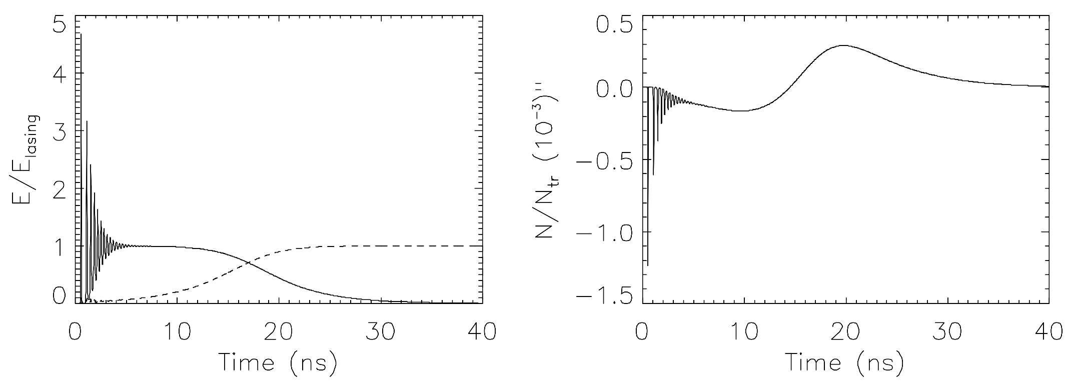

Fig. 1: Energy flow between modes (left figure) is indicated by non-zero carrier grating terms (right figure). The carrier grating terms can be monitored to indicate when multimode dynamics are present.

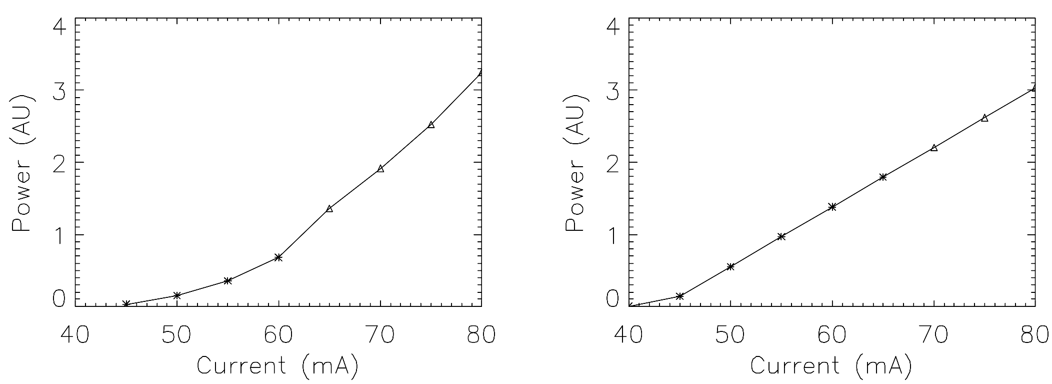

Fig. 2: Kink instability calculated with the PDE model (left figure) and 5 mode model (right figure) with the same parameters. The 5 mode model correctly indicates that multimode dynamics is important between 60 and 70 mA but does not accurately predict exactly where the kink occurs. The coupled mode model can quickly scan the current range and flag regions which need more careful study with the PDE model.