Simulating the Modulation Response of VCSEL's with MINILASE III

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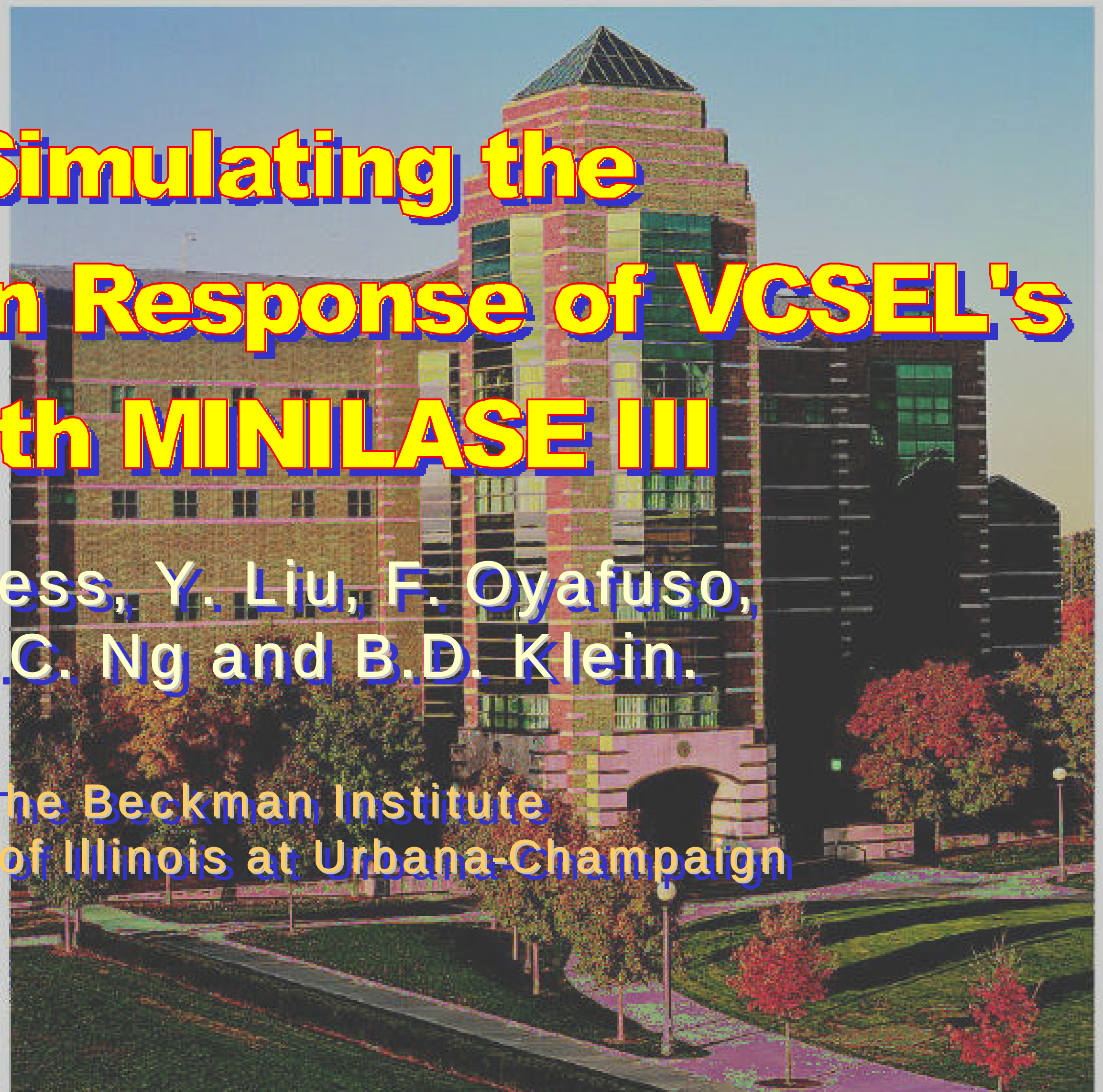
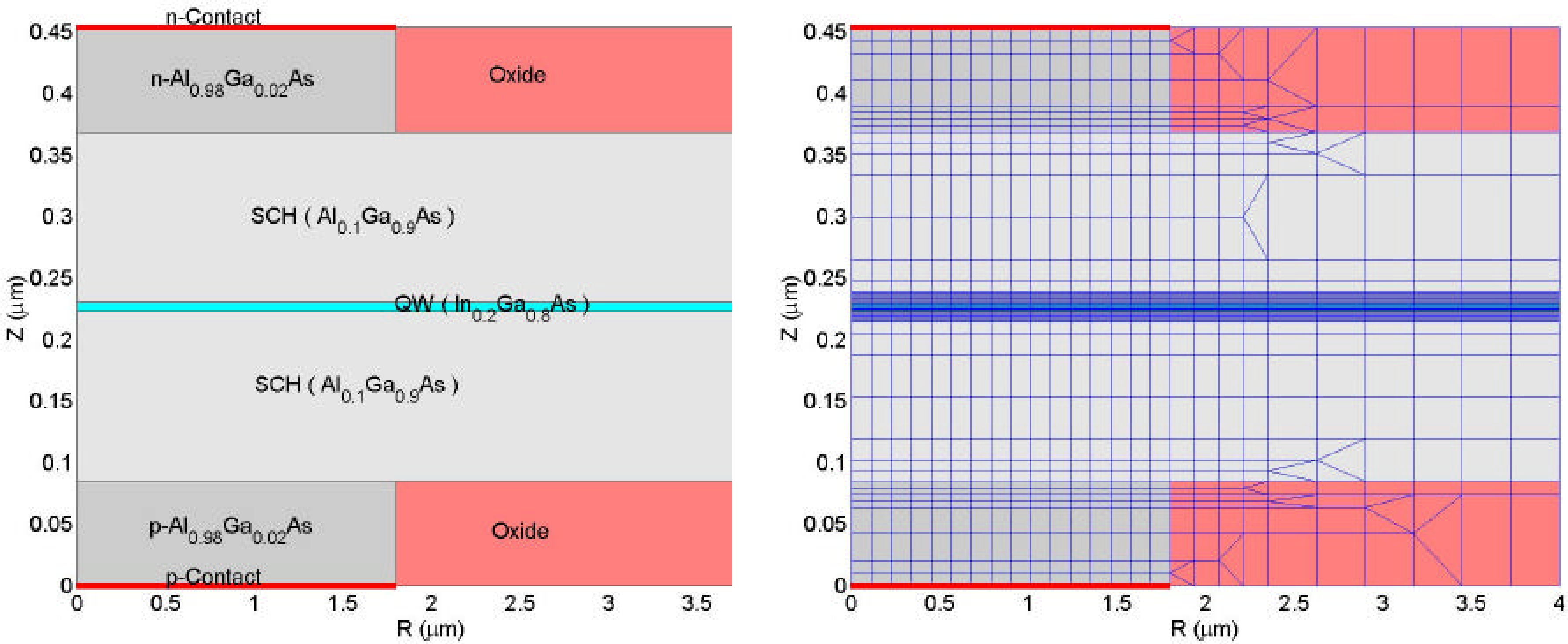


Photo by Don Hamerman

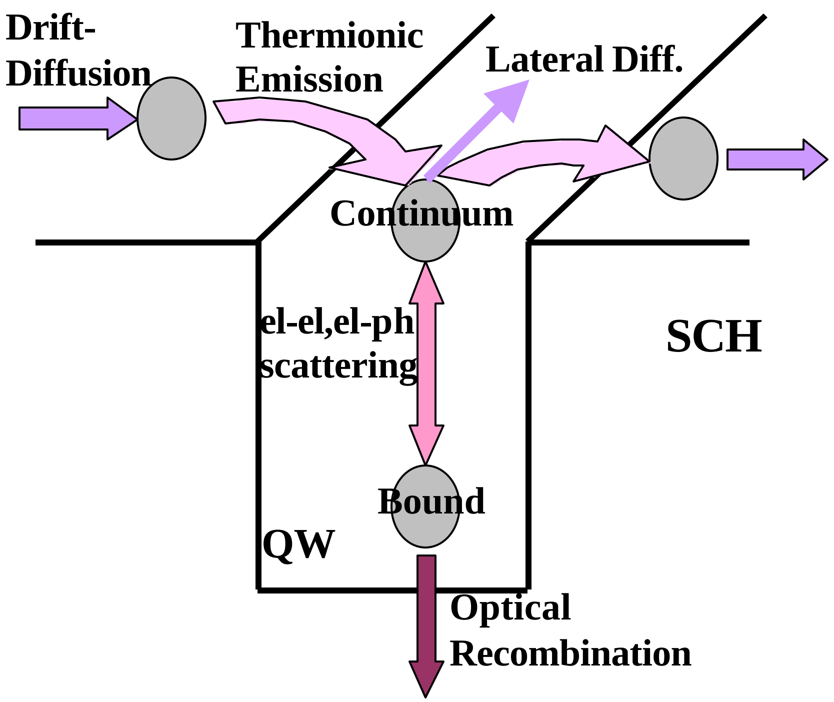
Typical VCSEL Structure and Electronic Mesh in MINILASE III



Electronic Solver in MINILASE III

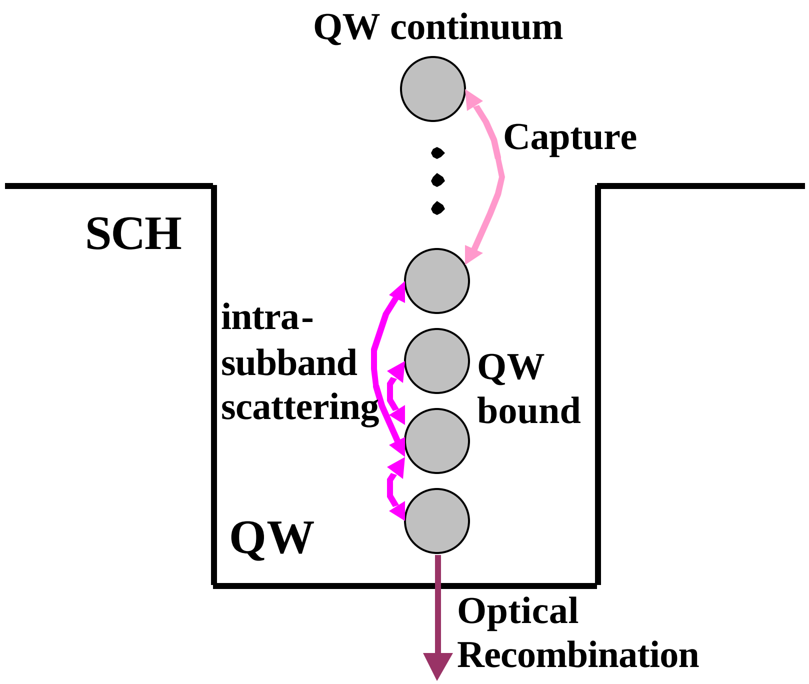
(Real Space Problem)

- Poisson Equation
- Continuity Equations for both electrons and holes
  - Drift diffusion in the bulk regions
  - Thermionic emission at the heterojunctions
  - QW carriers are divided into a continuum-state part and a bound-state part with different quasi-Fermi levels



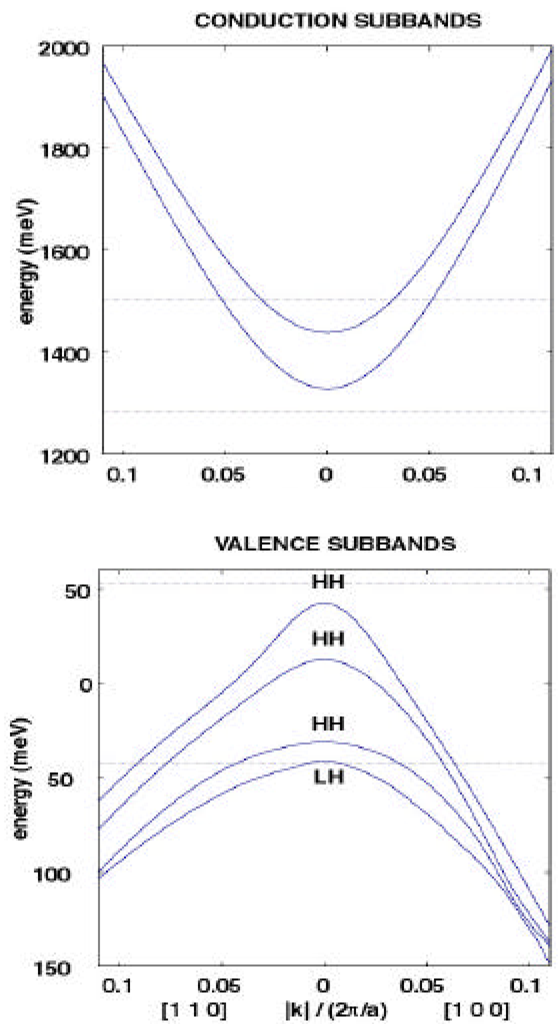
Electronic Solver in MINILASE III
(energy space problem)

- For QW carriers, the energy space is discretized
- To obtain the energy distribution, a 1D Boltzman equation is solved
- Both electron-electron and electron-phonon intra-subband scatterings are considered
- An additional phonon rate equation is solved to account for phonon decay



Electronic Solver in MINILASE III

(QW band structure)



- Eight-band k•p calculation of energy states in quantum well(s) including
  - Strain effects
  - Carrier-density-dependent exchange-correlation potential

- QW k•p states used to calculate optical matrix elements including Coulomb enhancement effect

Optical Solvers

(Green's Function Method)

Inhomogeneous Fredholm Integral Equation:

$$\mathbf{E}(\mathbf{r}) = \int d\mathbf{r}\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_{spon} + \int d\mathbf{r}\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot[i\omega\epsilon\chi_{gain}]\,\mathbf{E}(\mathbf{r}) + \int d\mathbf{r}\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot[i\omega\epsilon\chi_{oxide}]\,\mathbf{E}(\mathbf{r})$$

$$= \mathbf{S}_{spon} + \overline{\mathbf{L}}_{gain}\cdot\mathbf{E}(\mathbf{r}) + \overline{\mathbf{L}}_{oxide}\cdot\mathbf{E}(\mathbf{r}) \quad \text{(1)}$$

Corresponding Homogeneous Equation:

$$\mathbf{E}_n(\mathbf{r}) = \kappa_n\overline{\mathbf{L}}_{gain}\cdot\mathbf{E}(\mathbf{r}) + \overline{\mathbf{L}}_{oxide}\cdot\mathbf{E}(\mathbf{r}) \quad \text{(2)}$$

where $\mathbf{k}_n$ is a gain eigenvalue introduced to ensure a self-sustained lasing resonant mode.

- The dyadic Green's function, $\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')$, contains the response behavior of the planar homogeneous multi-layer open cavity of the VCSEL.

(Green's Function Method)

- Discretize only the Gain and Oxide regions.
- Use the Method of Moments to transform Eq. (2) into a generalized eigenvalue problem.
- Near the resonant frequency, Ω,

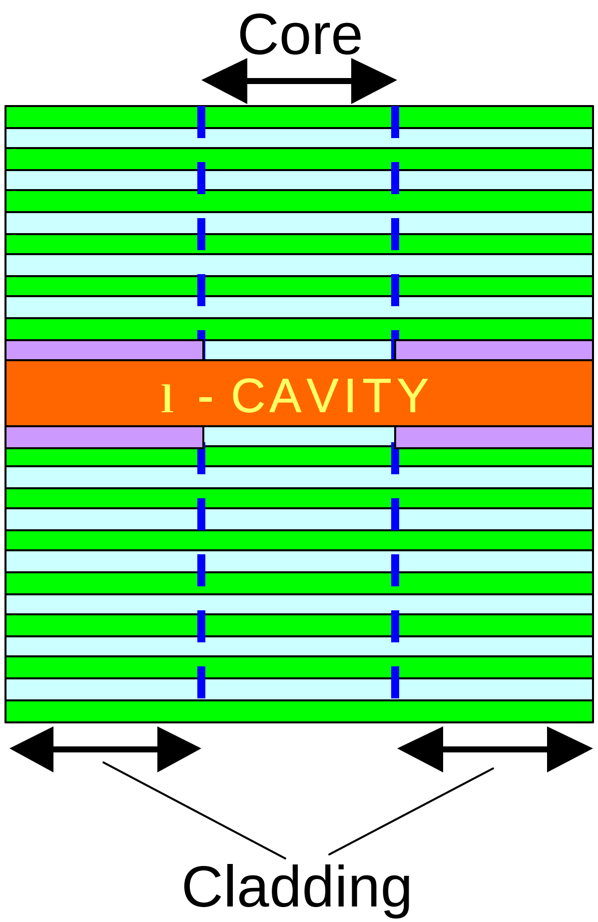
$$\kappa_n(\omega) \simeq \kappa_n(\Omega) + \left.\frac{\partial \kappa_n}{\partial \omega}\right|_{\Omega}(\omega - \Omega) \quad (3)$$

- Solve the generalized eigenvalue problem at a few frequencies, ω, to obtain a set of corresponding values for $\mathbf{k_n}$.
- At resonance, $\mathbf{k_n}$ must be real because the resonant field must be in phase with itself.
- The modal gain is then given by,

$$\frac{i\Delta G}{2} = \frac{1 - \kappa_n(\Omega)}{\left.\frac{\partial \kappa_n}{\partial \omega}\right|_{\Omega}} \quad (4)$$

Optical Solvers

(Weighted Index Method)



Scalar Wave Equation:

$$\nabla^2\phi + k^2\phi = 0 \qquad (5)$$

Assume Separable Wavefunction:

$$\phi(x,y,z) = \phi_s(x,y)\cdot\phi_z(z) \qquad (6)$$

Transverse- and z-wave Equations:

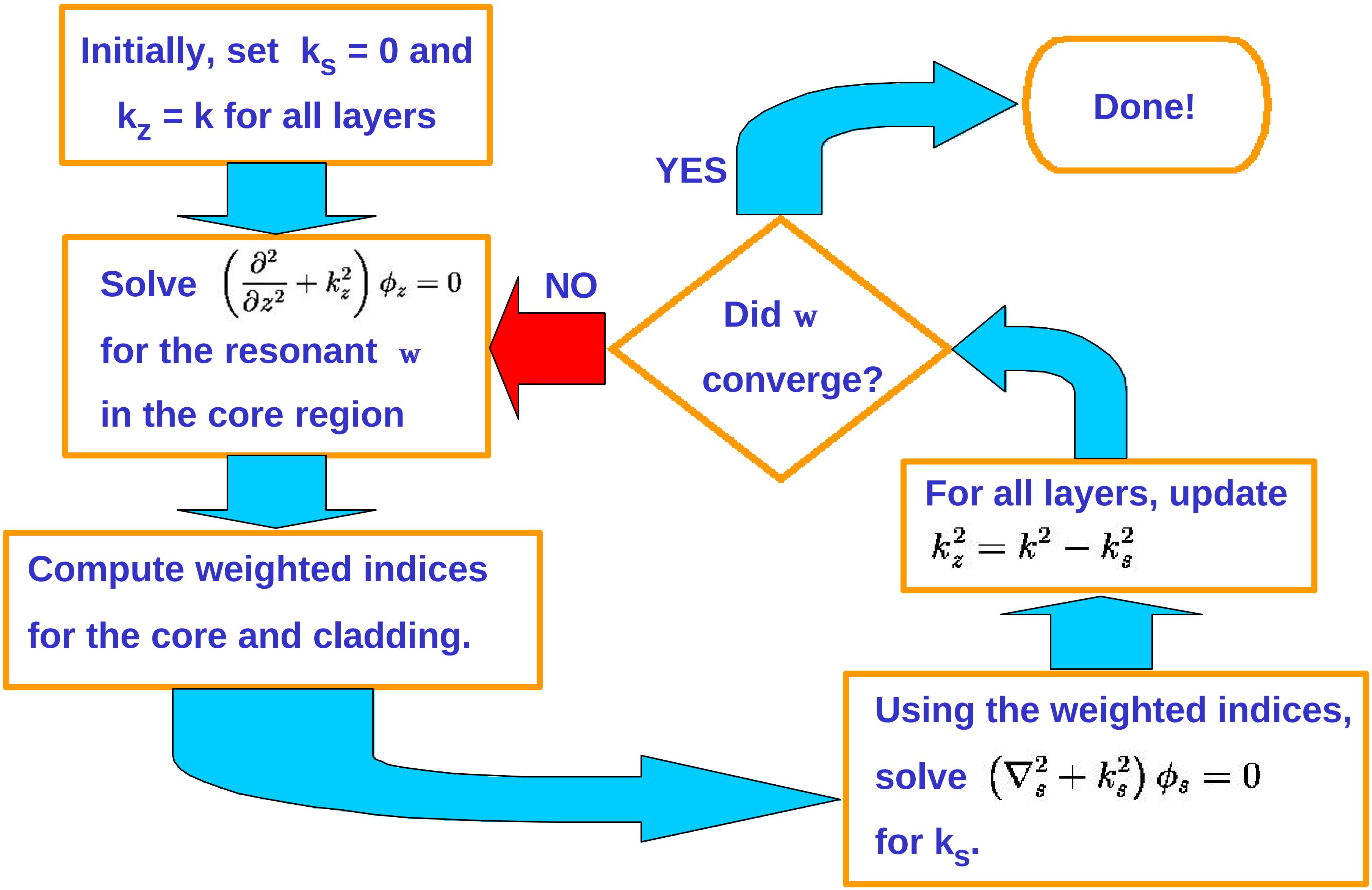
$$\nabla_s^2\phi_s + k_s^2\phi_s = 0 \qquad (7)$$

$$\frac{\partial^2}{\partial z^2}\phi_z + k_z^2\phi_z = 0 \qquad (8)$$

The dispersion relation:

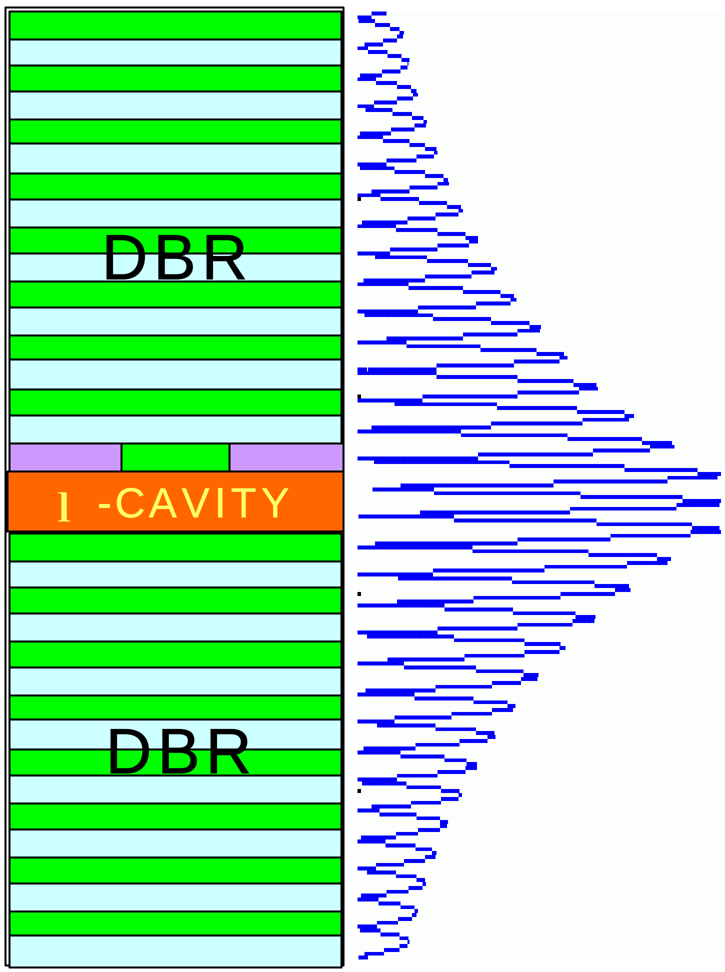
$$k^2 = k_s^2 + k_z^2 \qquad (9)$$

Algorithm for the Weighted Index Method

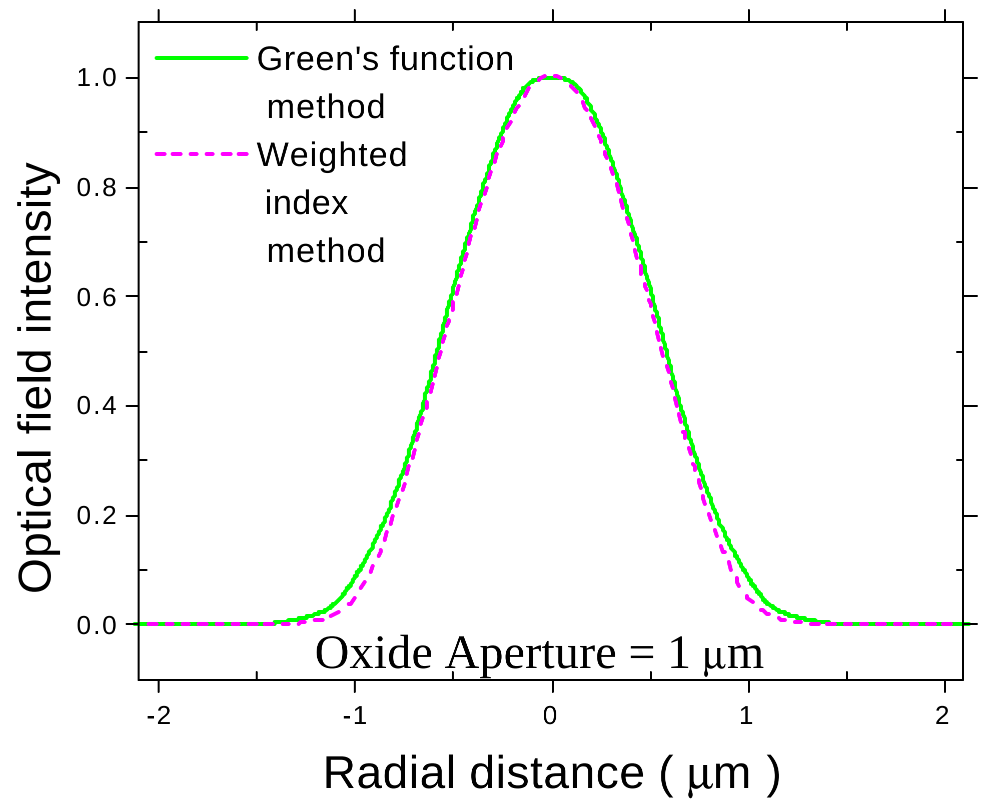


Resonant Field Profile

(Fundamental HE11 mode)

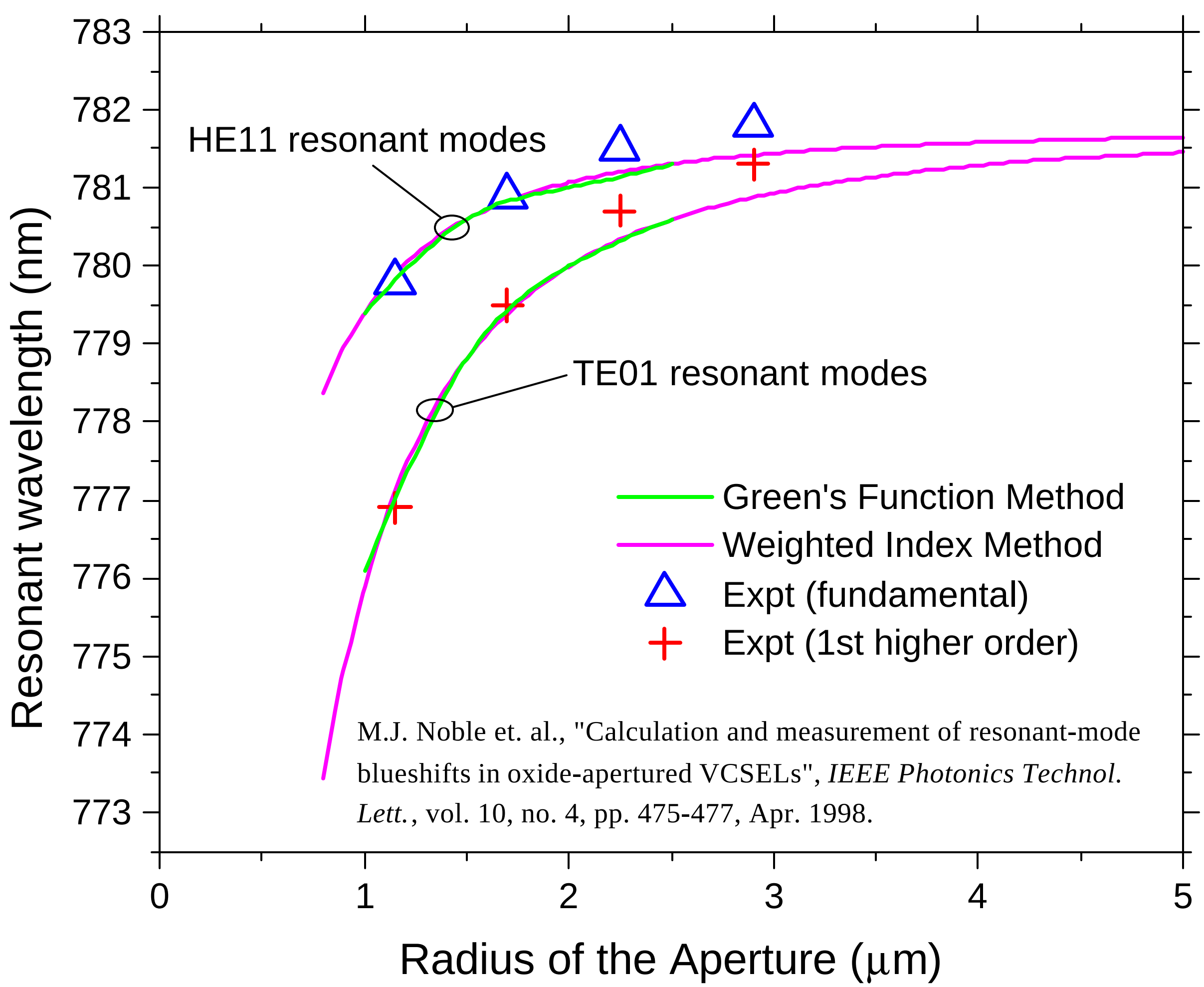


Vertical field variation

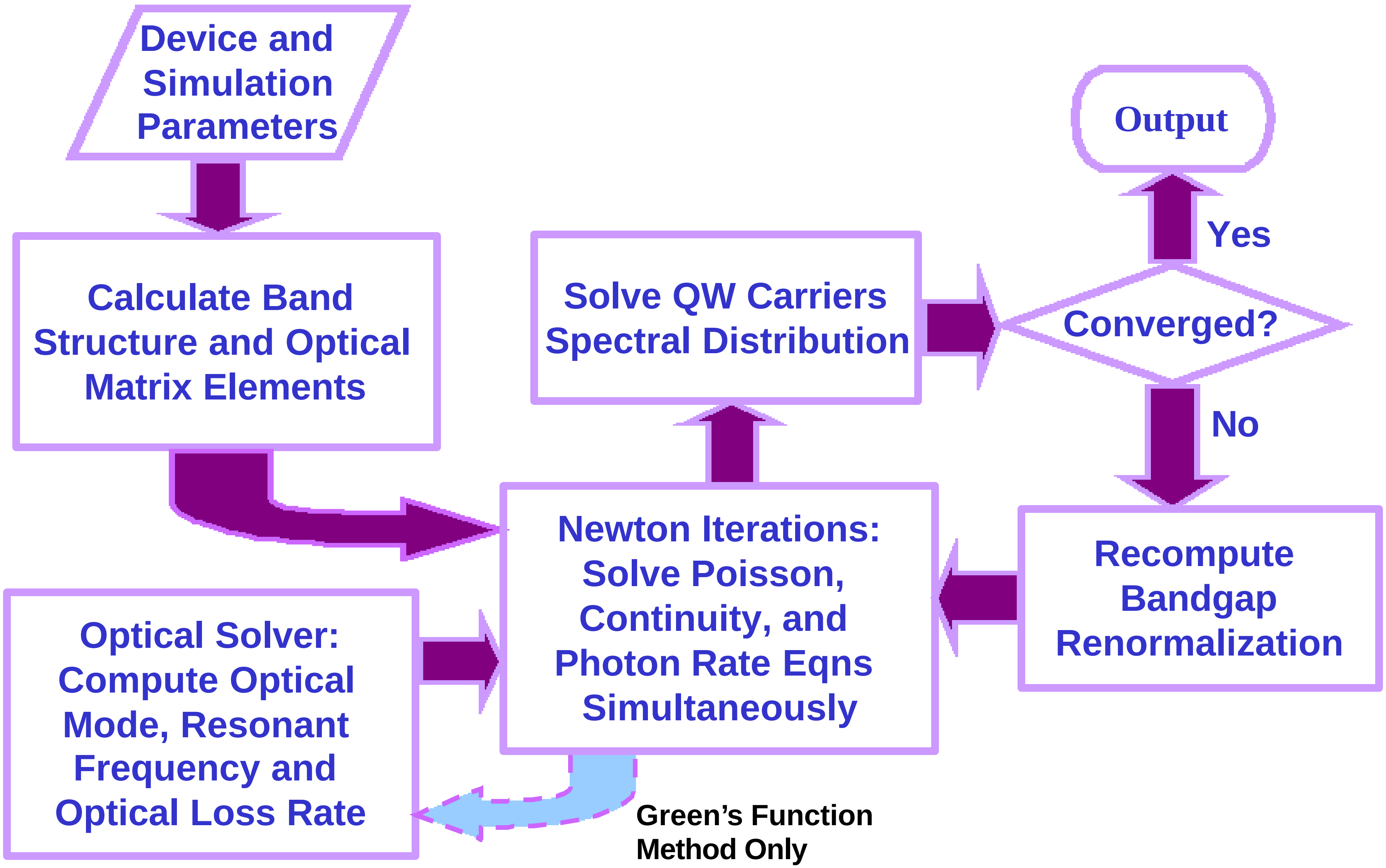


Lateral field variation in QW

Comparison of Resonant Wavelengths



Simulation Flow-Chart

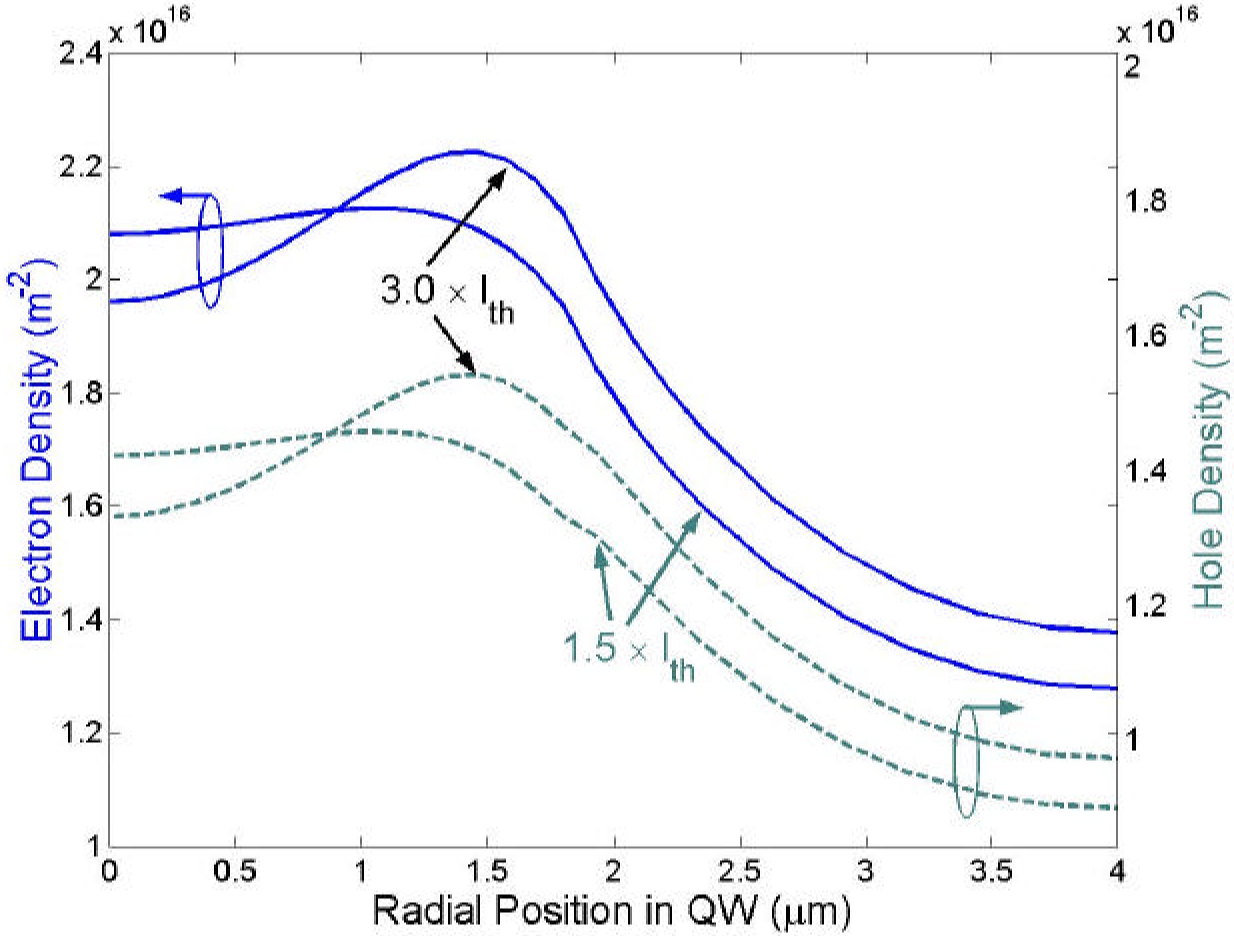


Simulation Flow-Chart (cont'd)

Within this self-consistent model, we can examine

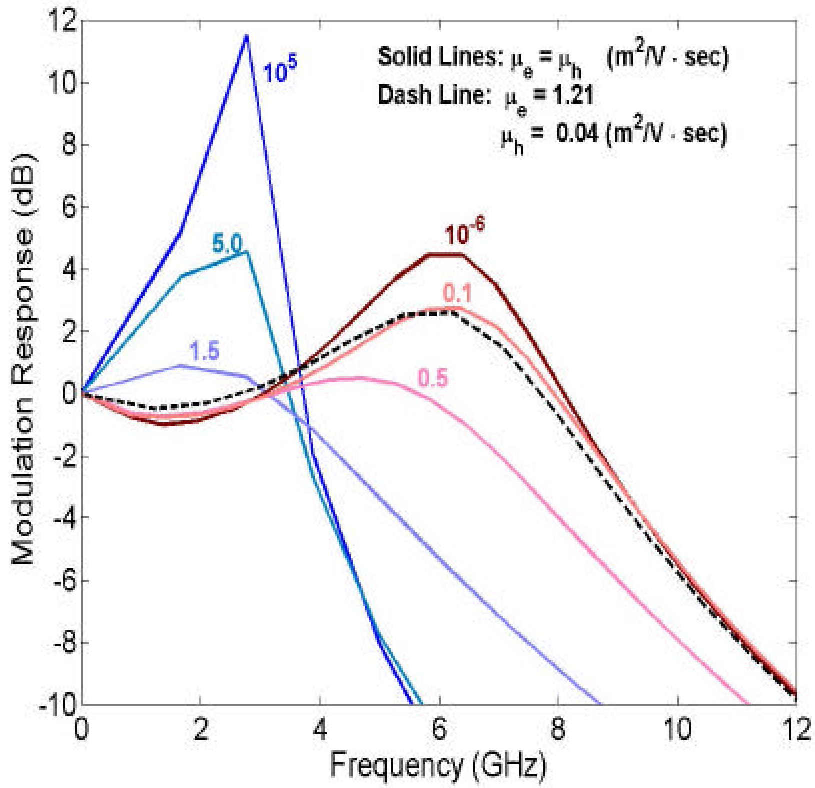
- Spatial hole burning and lateral transport effects
- Vertical transport effects and the diffusive capacitance caused by minority carriers
- Spectral hole burning and hot electron effects
- Effects of the oxide aperture size and more

Spatial Hole Burning for QW Electrons/Holes at Different Biases

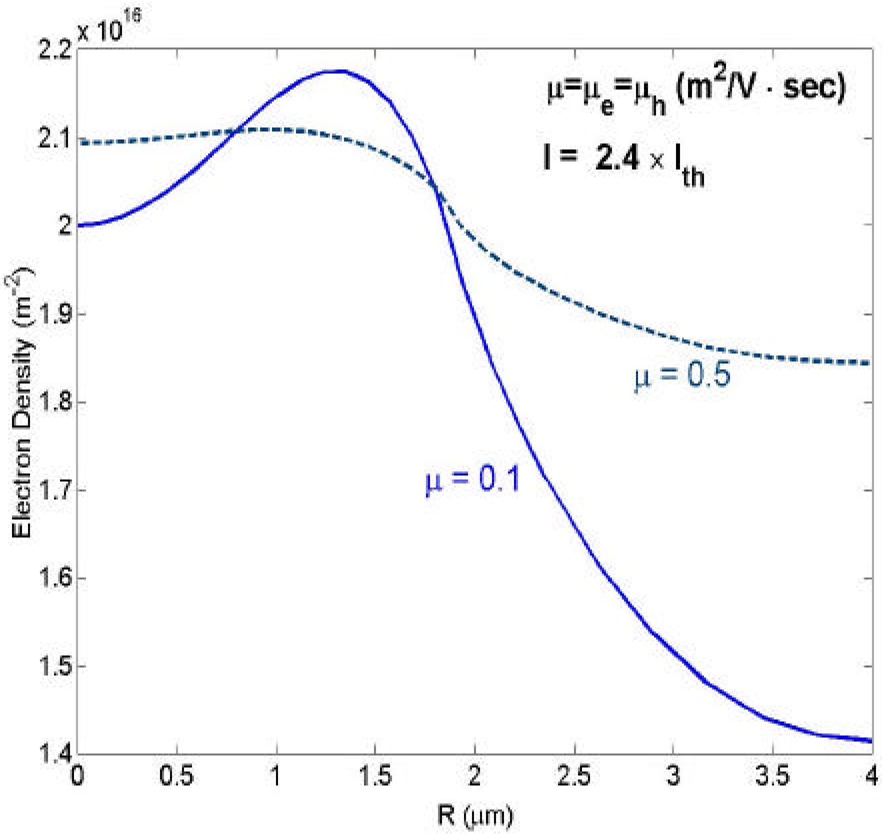


Effects of Lateral Diffusion in QWs on Modulation Responses

- Increasing mobilities reduces relaxation frequency.
- Relaxation peak is high for both high/low mobilities.
- Relaxation peak is dampened for moderate mobility values.
- Realistic values correspond to the low mobility limit, where the peak is dampened while the bandwidth is not reduced much.

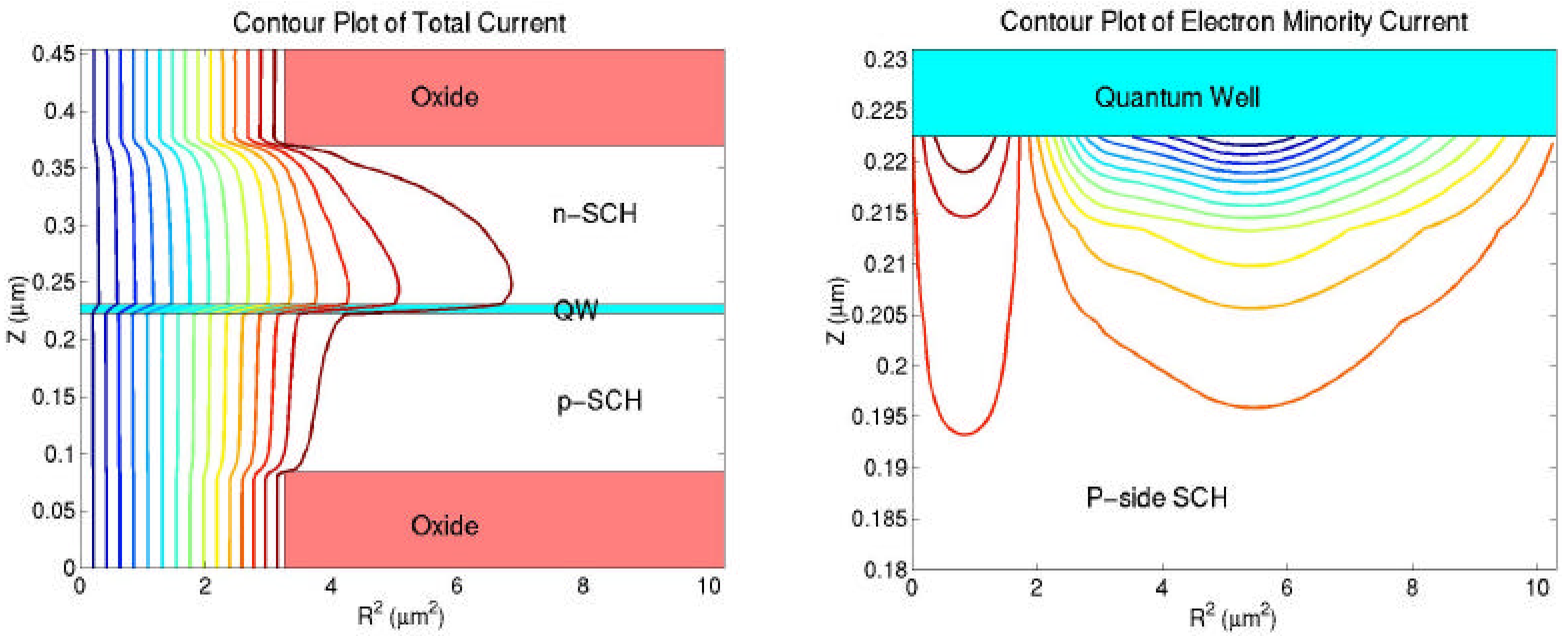


Explanation: Two Competing Mechanisms



- Due to small transverse mode size, stimulated recombination tends to burn a spatial hole at the center.
- Lateral diffusion tends to smooth out SHB.

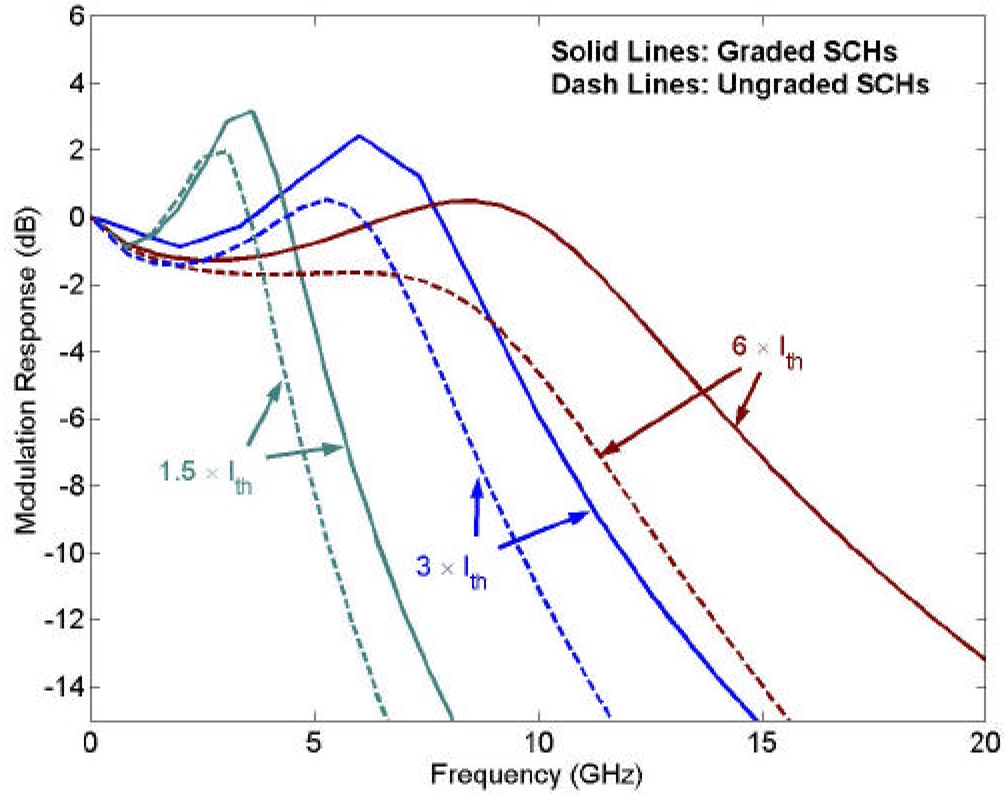
Typical Total Current and Minority Current Contours



- The minority carriers can be seen flowing back into the QW.
- Wasted minority carriers can introduce a vertical diffusive capacitance.

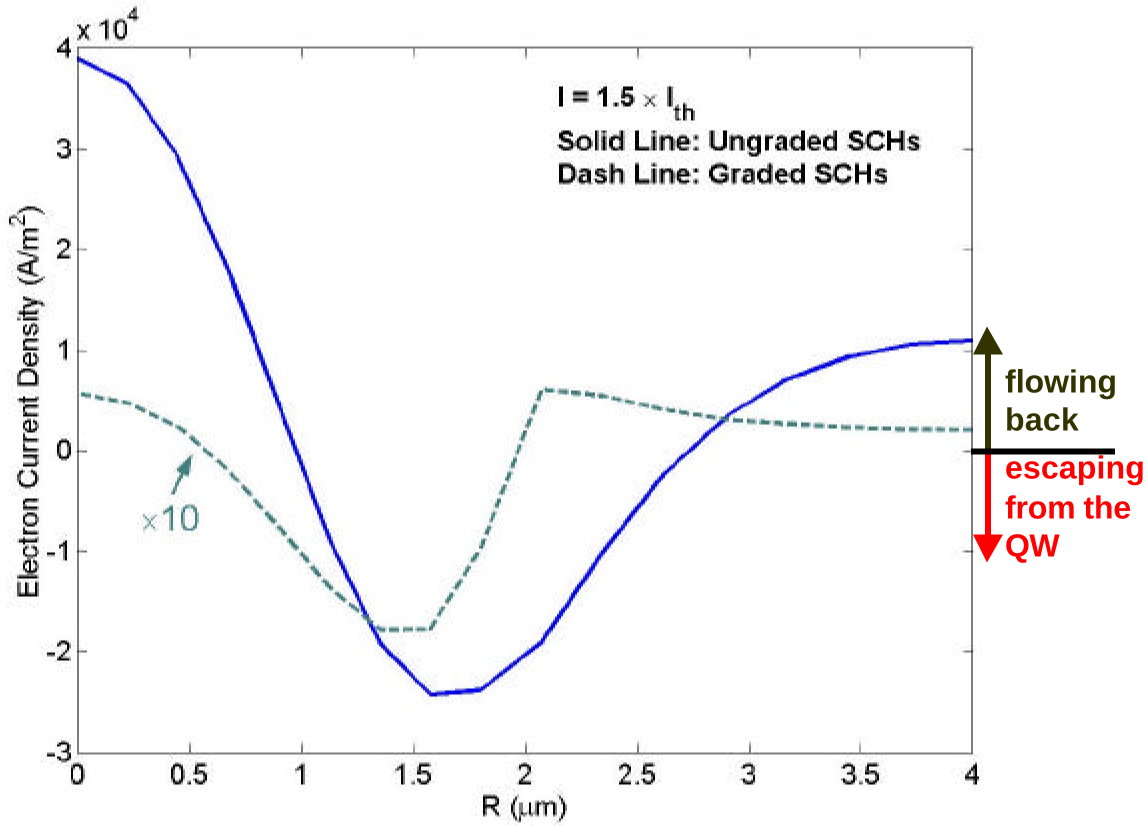
Mod. Resp. for "artificial" 2-λ VCSELs with Graded or Ungraded SCHs
(to demonstrate electrical transport effects)

- Grading the SCHs increases both the bandwidth and the height of the relaxation peak greatly.
- Smaller low frequency roll-off for graded VCSEL structure.



Current Density of Minority Charges Escaping from the QW

- Grading the SCHs reduces the minority current greatly (two orders of magnitude in this example)
- With graded SCHs the dynamic response can be improved due to suppressed diffusive capacitance



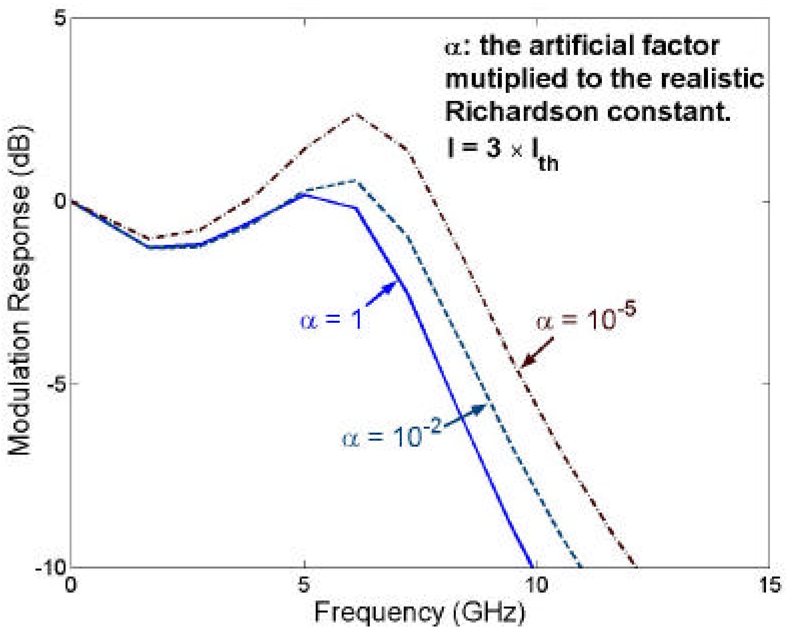
Further Study of Minority Carrier Effects

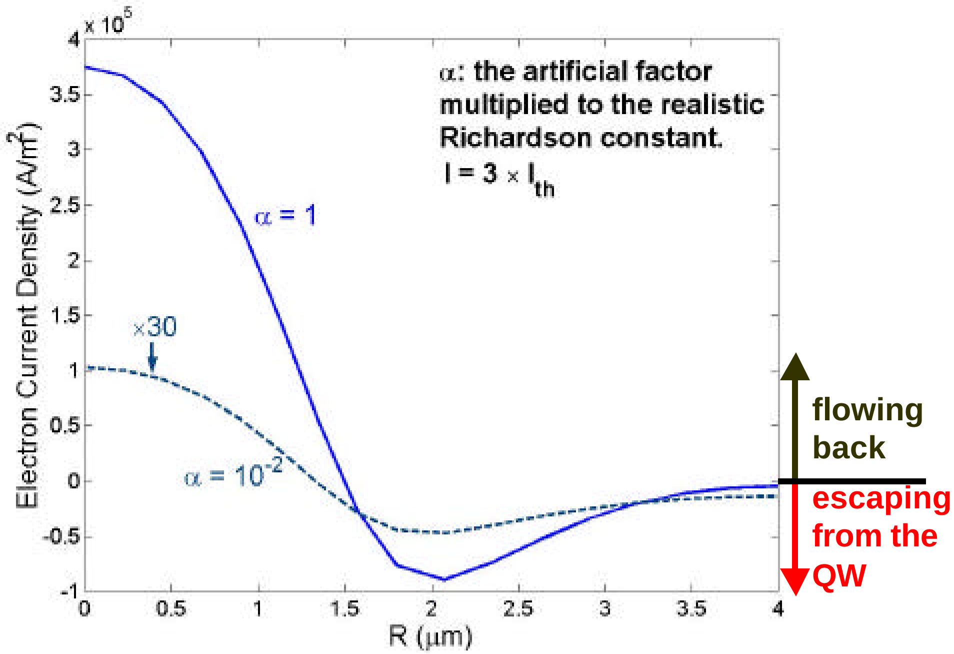
- At the heterojunctions, carrier fluxes are computed using Bethe's thermionic emission theory:

$$j_{\text{Á}\rightarrow\text{Â}} = \text{Á}^{*}\text{Ô}^{2}\left[\exp\left(\frac{F_{n,\text{Á}} - \text{Å}_{c,\text{Á}}}{k_B \text{Ô}}\right) - \exp\left(\frac{F_{n,\text{Â}} - \text{Å}_{c,\text{Â}} - \text{ÄÅ}_c}{k_B \text{Ô}}\right)\right]$$

- In MINILASE, the Richardson constant, A^{*}, at the minority side of the QW can be artificially reduced to suppress the minority current. This is equivalent to raising the barrier height to suppress carriers escape.

Effects of Artificially Reduced Richardson Constant





- Minority current is greatly suppressed.
- Similar improvements of modulation responses are observed.

Summary

- As a comprehensive first-principle tool, MINILASE III has been developed to account for nonlinear gain effects in VCSELs.
- With MINILASE III, spatial hole burning and carrier lateral transport effects on the modulation response of VCSELs are studied.
- Minority carriers are found to be responsible for a diffusive capacitance in the modulation response of VCSELs as shown by our simulations with MINILASE III.