Two Models for Electro-Magnetic Wave Amplifier by Utilizing Traveling Electron Beam.

by,

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1-General Scheme of Electro-Magnetic Wave Amplifiers.

2-Theoretical Models of Amplification Mechanism.

3-Thermal Effect on the Amplification Gain.

4-Experimental Evidences.
What is the Electro-Magnetic wave Amplifiers?
Electro-Magnetic (EM) wave Amplifiers

- Scheme of EM wave amplifier is same from microwave region to X-ray region.

\[ g \approx \frac{F(z_2)}{F(z_1)} \rightarrow \partial F(z) \partial z \]

\( F(z) \) is field amplitude in z-direction and \( T_z(x,y) \) is transverse field distribution.

- Energy exchange between electron beam and EM-wave.

If the electric field component

\[ E = F(z)T_z(x,y)e^{j(\omega t - \beta z)} \rightarrow (1) \]

F(z) is field amplitude in z-direction and \( T_z(x,y) \) is transverse field distribution.

The amplification gain \((g)\) is,

\[ \frac{\partial F(z)}{\partial z} = \frac{g}{2} F(z) \rightarrow (2) \]
Amplification Models

“How does the electron see the EM-wave”? 
How does the Electron see the Electromagnetic wave?

Form of the electron wave function:

\[ \varphi_n(r) = \frac{1}{\sqrt{l^3}} e^{jk_nz} \rightarrow (3) \]

\( k_n \): the electron wave number at \( n \)-th level.

\( l \): the coherent length of electron wave “electron size”.

\[ l = 40 \mu m^{[1]} \]

Models of amplification gain analysis:

1- Coherent Electron Wave Model ($l \gg \lambda$)

Electron wave size ($l$) $> \lambda$.

- Electron wave picture.
- Quantum mechanical trend.

2- Localized Electron Model ($l \ll \lambda$)

Electron wave size ($l$) $< \lambda$, electrons density modulation is shown.

- Electron particle picture.
- Classical mechanical trend.
First Model
“Coherent Electron Wave Model”
1- Physical interpretation of amplification

The electron transits basing on the rules of:

\[ E_b - E_a = \hbar \omega \] "Energy conservation"

\[ k_b - k_a = \beta \] "Momentum conservation"

- \( E_i \) is the electron energy at level \( i \) (\( i = a \) or \( b \) or \( c \)).
- \( k_i \) is the electron wave number at level \( i \).
- \( \omega \) is emitted EM-wave frequency.
- \( \beta \) is EM-wave propagation constant.

The condition of amplification

**Electric field**

**Electron wave at final level**

**Electron wave at initial level**

**Mixed electron wave**

Spatial variation coincidence corresponds to momentum conservation
2- Gain coefficient in CEW-Model

By some tools of statistical quantum mechanics,

$$g \propto \left| \langle \phi_a | T(x, y) e^{-i\beta z} | \phi_b \rangle \right|^2 \rightarrow (4)$$

Finally, the expression of amplification gain in CEW-Model

$$g(v_b, v_{em}) = \sqrt{\frac{\mu_o}{\varepsilon_o}} \frac{e J_0 \tau v_b}{n_{eff} \hbar \omega} \xi \times D(v_b, v_{em}) \rightarrow (5)$$

$$\xi = \int_s |T_z(x, y)|^2 dx \, dy$$
(Coupling coefficient)

$v_b$ is electron velocity influenced by applied voltage $V_b$.
$v_{em} = c/n_{eff}$ is EM-wave phase velocity.
$J_0$ is average electron beam current density and $\tau$ is electron relaxation time.

$D$ is the dispersion function controlling the gain profile,

$$D(v_b, v_{em}) = \text{Sinc}^2 \left[ \left\{ \frac{2m_o}{\hbar} \left( \sqrt{eV_b} - \sqrt{eV_b + \hbar \omega} \right) - \frac{n_{eff} \omega}{c} \right\} \frac{\ell}{2} \right]$$

$$- \text{Sinc}^2 \left[ \left\{ \frac{2m_o}{\hbar} \left( \sqrt{eV_b + \hbar \omega} - \sqrt{eV_b} \right) - \frac{n_{eff} \omega}{c} \right\} \frac{\ell}{2} \right] \rightarrow (6)$$
Gain behavior with frequency variation in CEW-Model.

The gain peak is affected by saturation of the dispersion function.

\[ g(\nu_b, \nu_{em}) = \sqrt{\frac{n_{eff}}{\varepsilon_0 \eta_0}} \frac{\mu_o e J_o \tau \nu_b}{h \omega} \xi \times D(\nu_b, \nu_{em}) \]

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Variation of gain peak with EM frequency

Saturation of dispersion function to 1.
Second Model
“Localized Electron Model”
One synchronize wave modulates the electron velocity to start the amplification.
2- Gain coefficient in LE-Model

From the quantum mechanics point of view,

\[ \tilde{\Psi} = \sum_v C_v \Psi_v \rightarrow (7) \]

(Total wave function)

\( \Psi_v \) is the wave function of \( v \)-electron

The form of velocity-modulation, (same as classical form)

\[ \frac{\partial v_v}{\partial t} + \vec{v}_v \frac{\partial v_v}{\partial z} = - \frac{e}{m_o} \left\{ F(z)T_z(x, y) e^{j(\omega t - \beta z)} + c.c \right\} - \frac{v_v - \bar{v}_v}{\tau} \rightarrow (8) \]

Finally, The expression of amplification gain in LE-Model

\[ g(v_b, v_{em}) = \xi \frac{e\mu_o J_o}{m_o} \times Y(v_b, v_{em}) \rightarrow (9) \]

\( Y(v_b, v_{em}) \) is dispersion function controls gain profile,

\[ Y(v_b, v_{em}) = \text{Re} \left\{ \left( \frac{j + \frac{1}{\alpha \tau}}{\sqrt{\left( \frac{n_{\text{eff}}}{c} v_b - 1 + \frac{j}{\omega \tau} \right)^2}} \right) \right\}, v_{em} = \frac{c}{n_{\text{eff}}} \rightarrow (10) \]
Gain behavior with frequency in Localized Electron Model

The gain increases infinitely with frequency, thermal effect limits this behavior.

\[
g(v_b, v_em) = \xi \frac{e \mu_o J_o}{m_o} \times Y(v_b, v_em)
\]

Dispersion function in gain coefficient by the LE-Model.

Variation of gain coefficient with the EM frequency by the LE-Model.
Thermal Effect on the Amplification Gain
Real gain with thermal effect “velocity broadening around the average value”,

\[ g(\bar{v}, v_{em}) \approx \int_0^\infty f(v_b, \bar{v}) \ g(v_b, v_{em}) \ dv_b \rightarrow (11) \]

\[ f(v_b, \bar{v}) \] is the normalized Maxwell-Boltzmann distribution function.

Where,

\[
  f(v_b, \bar{v}) = \sqrt{\frac{m_o}{2\pi K_B T}} \exp \left[ -\frac{eV_b}{K_B T} \left( \frac{\bar{v}}{v_b} - 1 \right)^2 \right]
\]

\( \bar{v} \) is the average electron velocity and \( v_b \) is the real electron velocity.

\( K_B \) is the Boltzmann constant and \( T \) is the absolute temperature.

Illustration of thermal distribution function.
The effect of thermal velocity broadening on gain amplification.

\[ g(\overline{v}, v_{em}) \approx \int_{0}^{\infty} f(v_{b}, \overline{v}) g(v_{b}, v_{em}) \, dv_{b} \]

The boundary between two models within THz region.

Variation of the peak values of gain coefficient with EM frequency by the CEW-Model and the LE-Model for several temperatures.
The effect of electron size on the gain amplification

The thermal effect gives same gain peaks for different coherence length.

Gain with different assumed coherence length at different temperature.
Experimental Setup

- Focusing lens
- InGaAs
- Photo-detector
- Laser Diode Current
- Driver
- Experiment Setup
- Function Generator
- Magnetic lens
- SOI
- Vacuumed Chamber
- Monochromator
- Lock. In AMP
- Electron beam SIO
- SiO2
- 0.32 μm
- 1 μm
- 100 μm
- Si
- Computer screen
- “Signal Express software”
- Computer Interface
- Electron gun
- Switch
- Laser Diode Current Driver
- Function Generator
- InGaAs Photo-detector
- Experiment Setup
Comparison of the emission profile with theoretical Calculation

\[ g(v_b, v_{em}) \propto \text{Sinc}^2 \left[ \frac{\sqrt{2m_0}}{\hbar} \left( \sqrt{eV_b - \sqrt{eV_b - \hbar \omega}} - \frac{n_{\text{eff}} \omega}{c} \right) \ell \right] \]

**EMISSION**

\[ \text{Sinc}^2 \left[ \frac{\sqrt{2m_0}}{\hbar} \left( \sqrt{eV_b - \sqrt{eV_b - \hbar \omega}} - \frac{n_{\text{eff}} \omega}{c} \right) \ell \right] \]

**ABSORPTION**

\[ \frac{1}{N^{1/3}} \]

\( \ell = \frac{1}{N^{1/3}} \)

*\( N \) is electrons density*
Emission spectrum for different acceleration voltage

\[ v_b (\text{[ at synchronization]}) = \frac{I}{\hbar} \frac{\partial E}{\partial k} = \frac{I}{\hbar} \frac{E_b - E_a}{k_b - k_a} \]

\[ = \frac{\omega}{\beta(\lambda)} = v_{em} \]
THANKS FOR YOUR ATTENTION