Space Harmonic Distribution at Bragg Condition in Periodical Dielectric Waveguides

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Abstract—By Floquet-Bloch theory, the electric and magnetic fields are the summation of an infinite number of space harmonics, where the harmonics include forward and backward propagated harmonics. In this paper, the distribution of space harmonics are analyzed at the Bragg condition in grating structure of dielectric waveguides.

I. Introduction

Contra-directional mode coupling in periodic dielectric waveguides is important in distributed Bragg reflector (DBR) lasers and distributed feedback (DFB) lasers. The geometry of the waveguide structure consists of a surface corrugation and two uniform dielectric waveguides. Figure 1 illustrates the basic geometry of the grating layer with a grating section. In this paper, we demonstrate the power distribution in the periodical waveguide structure. The Floquet-Bloch theory [1] and the Mahmoud-Beal’s are applied to calculate the transmission and reflection spectra of the grating structure [2].

II. PROBLEM FORMULATION

In the paper, the Floquet-Bloch theory (FBT) is used to analyze the periodical waveguide structure [1]. According to the Floquet-Bloch theory, the field pattern should consist of an infinite number of space harmonics. For the sake of simplicity it is assumed that the field is invariant with respect to \( y \). Since the field is assumed to propagate in the \( z \)-direction, the solution of the Floquet-Bloch theory can be expressed as

\[
E_y(x,z) = \sum_{n=-\infty}^{\infty} f_y(n) \cdot \exp(-jk_{zn}z) = E_{yf} + E_{yb} \quad (1)
\]

And

\[
E^b_y(x,z) = \sum_{n=-\infty}^{\infty} f^b_y(n) \cdot \exp(jk_{zn}z) = E_{yf}^b + E_{yb}^b \quad (2)
\]

where

\[
k_{zn} = \beta_n + j\alpha = (\beta_0 + nk) + j\alpha, \quad \alpha < 0, \quad K = \frac{2\pi}{\Lambda}
\]

Here \( K \) is the grating wavenumber, \( n \) is the space harmonic order, and \( i \) represents the \( i \)th layer. The values \( k_{zn} \) is the complex propagation constant of the \( n \) th spatial harmonic. \( \beta \) is the propagation constant, \( \alpha \) indicates attenuation constant. The real part of \( k_{zn} \), \( f_n \), is the phase constant of the \( n \)th space harmonic, and the imaginary part of \( k_{zn} \), \( \alpha \) (\( \alpha < 0 \)), is the attenuation constant due to the leakage of the guided-wave energy into the substrate and the superstrate regions. As expressed in (1) and (2), electric fields in the grating region consist of forward harmonics \( E_{yf} \) and \( E_{yf}^b \) and backward harmonics \( E_{yb} \) and \( E_{yb}^b \). Consider the vicinity of the first Bragg region. \( E_{yf} \), \( E_{yb} \), \( E_{yf}^b \) and \( E_{yb}^b \) can be written as

\[
E_{yf} = \sum_{n=0}^{\infty} f_y(n) \cdot \exp(-jk_{zn}z) \quad (3)
\]

\[
E_{yb} = \sum_{n=-\infty}^{-1} f_y(n) \cdot \exp(-jk_{zn}z) \quad (4)
\]

and

\[
E_{yf}^b = \sum_{n=0}^{\infty} f_y(n) \cdot \exp(jk_{zn}z) \quad (5)
\]

\[
E_{yb}^b = \sum_{n=-\infty}^{-1} f_y(n) \cdot \exp(jk_{zn}z). \quad (6)
\]

Figure 1 shows that the fundamental mode (\( e_y \)) is launched at region I towards the grating. This produces the reflected and transmitted fields. As shown in Fig. 1, \( T_1 \) and \( R_1 \) are the transmission and reflection coefficients in the grating region, while \( t \) is the transmission coefficient in region II, and \( r \) is the reflection coefficient in region O. The coefficients \( T_1, R_1, \) and \( t \), \( r \) are determined from the boundary conditions at the input and output planes of the grating.

![Fig. 1 The basic geometry of the two grating sections.](image)

For the uniform plane wave, the boundary condition at the interference \( z=0 \) can be expressed as
In dielectric waveguide structure, the relationship between \( r \), \( T_1 \), and \( R_1 \) is obtained by overlapping the boundary condition at non-grating and grating layer interfaces. The overlap equations are expressed as

\[
e_j^L + r \cdot e_j^n = T_1 \left( E_{y,1f} + E_{y,1b} \right) + R_1 \left( E_{y,1f}^b + E_{y,1b}^b \right)
\]

(7)

\[
h_j^L - r \cdot h_j^n = T_1 \left( H_{x,1f} + H_{x,1b} \right) - R_1 \left( H_{x,1f}^b + H_{x,1b}^b \right)
\]

(8)

In this paper, the reflection and the transmission efficiencies of periodical dielectric waveguide in a grating region are obtained by using the Mahmoud-Beal’s method and the FBT.