# Modeling of Semiconductor Optical Amplifier RIN and Phase Noise for Optical PSK Systems

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Abstract- Phase modulation schemes are attracting much interest for use in ultra-fast optical communication systems because they are much less sensitive to fibre nonlinearities compared to conventional intensity modulation formats. Semiconductor optical amplifiers (SOAs) can be used to amplify and process phase modulated signals, but with a consequent addition of nonlinear phase noise (NLPN). Existing SOA NLPN models are simplistic. In this paper we show that a more accurate model can be used, which results in simple expressions for SOA nonlinear noise, in particular when used to amplify differential phase shift keyed (DPSK) modulated data. The model is used to calculate the optical signal to noise ratio introduced by a power booster SOA and the first inline amplifier of a 40 Gb/s NRZ-DQPSK single channel link.

#### I. INTRODUCTION

\*Constant envelope modulation formats, in particular RZand NRZ-DPSK, are among the most promising candidates for SOA-based high bit rate systems because of their resilience to fiber non-linearities and pattern effects [1]. Gain saturation in SOAs introduces NLPN that can be detrimental in PSK systems. The phase noise behaviour of saturated SOAs in DPSK systems has been analysed in several papers [2 and its references]. The advantage of these NLPN models is that they are analytical and easy to apply, however they have limited accuracy, in that they do not consider the internal noise generated by the SOA or properly account for scattering losses. In this paper we show that these assumptions are not always correct. We also show that an existing computationally simple and more accurate noise model can be used that leads to simple NLPN expressions at the SOA for constant envelope modulation schemes [3].

## II. THEORY

The model equations in the spectral domain ( $\omega$ ) are [3]:

$$\frac{\partial}{\partial z} \frac{\delta \rho(z)}{\rho_{S}(z)} = -\frac{g_{S}(z)\rho_{S}^{2}(z)}{1 + \rho_{S}^{2}(z) + i\omega\tau} \frac{\delta \rho(z)}{\rho_{S}(z)} + N_{\rho}(\omega, z)$$
(1)

$$\frac{\partial \delta\phi}{\partial z} = \frac{\alpha g_{S}(z)\rho_{S}^{2}(z)}{1 + \rho_{S}^{2}(z) + i\omega\tau} \frac{\delta\rho(z)}{\rho_{S}(z)} + N_{\phi}(\omega, z)$$
(2)

where z is the distance from the SOA input,  $\rho_S(z)$  is the signal field amplitude (equal to the square root of the optical power to the SOA saturation energy),  $\delta\rho(z)$  the amplitude noise (induced by the Langevin force  $N_\rho(\omega,z)$ ),  $\delta\phi(z)$  the phase noise (induced by the Langevin force  $N_\phi(\omega,z)$ ),  $g_S(z)$  the saturated gain,  $\tau$  is the carrier lifetime and  $\alpha$  the linewidth enhancement factor. The Langevin forces account for field fluctuations due to spontaneous emission, carrier noise and a term arising from their interaction. The terms  $\rho_S(z)$  and  $g_S(z)$  are the unperturbed, distributions along the SOA and can be obtained by numerically solving [3],

$$\frac{\partial E(z)}{\partial z} = \frac{1}{2} g_s(z) (1 - i\alpha) - \gamma_{SC} E(z)$$
(3)

$$0 = \left[ g_o - g_S(z) \right] / \tau - g_S(z) \left| E(z) \right|^2 / \tau \tag{4}$$

$$E(z) = \rho_{S}(z) \exp[i\phi_{S}(z)]$$
(5)

where  $\phi_s(z)$  is the optical field phase,  $g_o$  is the unsaturated gain, and  $\gamma_{sc}$  the waveguide scattering losses. Once the spatial dependency of  $g_S(z)$  and  $\rho_S(z)$  have been determined, they can be inserted in (1-2), which are then numerically solved to determine the SOA output RIN and phase noise, due to the noise of the input signal, spontaneous emission noise, carrier noise and the cross-correlation between the carrier noise and spontaneous emission. The numerical solution of (1-2) is not difficult as it mainly involves numerical integration.

## III. NUMERICAL ANALYSIS

The geometrical and material parameters used in the model were determined for a 1 mm long tensile-strained SOA [4] with a 20 dB unsaturated gain,  $\alpha$ = 2.5, saturation power of 1.9 mW,  $g_0$  = 9500 m<sup>-1</sup> and  $\gamma_{sc}$  = 4500 m<sup>-1</sup>. Simulation results for SOA output RIN and phase noise for an input signal power normalised (to the saturation power) of 0.1, with no input RIN is shown in Figs. 1-2. The phase noise spectrum can be used to determine the phase noise variance  $\sigma_{\phi}^2$  after optical reception. For example if we consider a 40 Gb/s NRZ-DQPSK receiver, its bandwidth is typically 20 GHz.  $\sigma_{\phi}^2$  can be determined by obtaining the autocorrelation function  $R(\tau)$  of the filtered phase noise by taking its inverse Fourier transform and letting  $\tau$ = 0.

<sup>\*</sup> This research was supported by the Spanish Department of Education under the Programa Nacional de Recursos Humanos del Plan Nacional de I+D+I 2008-2011 and Science Foundation Ireland Investigator Grant 09/IN.1/I2641.

It is simple to carry out this numerically. In Fig. 3,  $\sigma_{\phi}$  is plotted versus the normalised input power. It can be seen that  $\sigma_{\phi}$  is quite sensitive to the degree of saturation. To our knowledge the probability density function of NRZ-DQPSK random data in the presence of NLPN has not yet been determined and so it is not presently possible to obtain an analytical expression for the bit-error-rate. However it is possible to determine the Optical Signal-to-Noise Ratio (OSNR) which is given by

$$OSNR = \pi^2 / 8\sigma_{\phi}^2 \tag{6}$$

In Fig. 4, the OSNRs at the outputs of a power booster (with no input noise) and a first in-line SOA in a 40 Gb/s DQPSK link are shown as a function of the SOA carrier lifetime. Both SOA input normalised powers are equal to 0.01. It can be seen that there is a serious degradation in the OSNR, particularly at the in-line SOA output, for small carrier lifetimes. Similar simulations show a similar dependency for increases in the linewidth enhancement factors and scattering losses.

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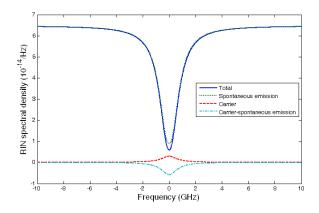


Fig. 1: SOA output total RIN and components spectra (centred at the optical frequency) for a normalised input power of 0.1.

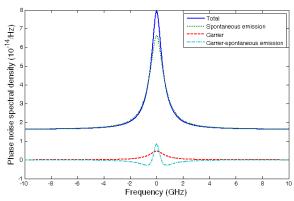


Fig. 2: SOA output total phase noise and components spectra (centred at the optical frequency) for a normalised input power of 0.1.

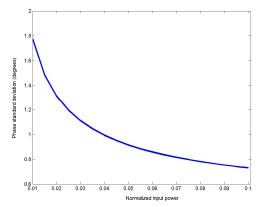


Fig. 3: SOA output phase noise standard deviation for input powers ranging from 0.01 to 0.1. The receiver bandwidth is 20 GHz.

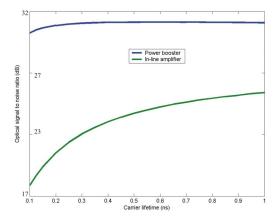


Fig. 4: OSNR at the output of SOA power booster and the first in-line amplifier as a function of the carrier lifetime for a normalised input power = 0.01. The receiver bandwidth is 20 GHz.